Electron-Ion Collisions
and
the Low-x Structure of Matter
(Theory)

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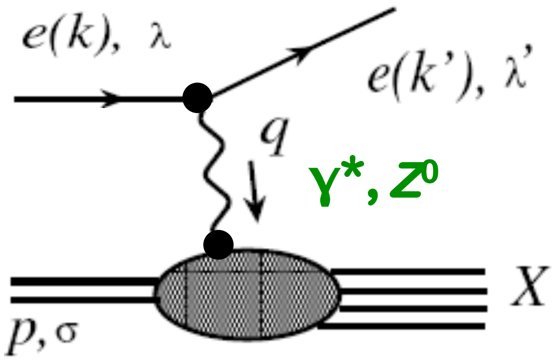
EIC Collaboration meeting
Lawrence Berkeley National Laboratory, CA, December 11-13, 2008

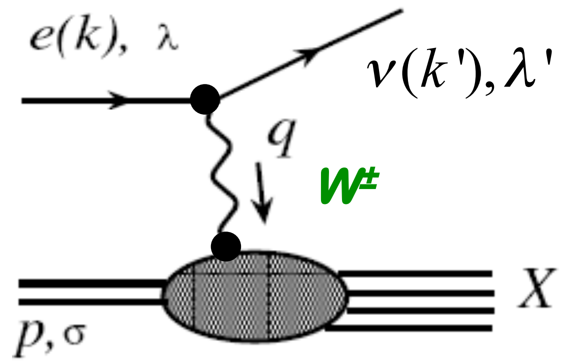
Inclusive DIS in ep and eA Collisions

❑ Inclusive DIS cross section:

Neutral current (NC)



Charged current (CC)



❑ Kinematic variables:

✧ **4-momentum transfer:** $Q^2 = -q^2$ ✧ **Bjorken variable:** $x_B = \frac{Q^2}{2p \cdot q}$

✧ **Squared CMS energy:** $s = (p+k)^2 = \frac{Q^2}{x_B\, y}$ ✧ **Inelasticity:** $y = \frac{p \cdot q}{p \cdot k}$

✧ **Final-state hadronic mass:** $W^2 = (p+q)^2 \approx \frac{Q^2}{x_B}(1 - x_B)$

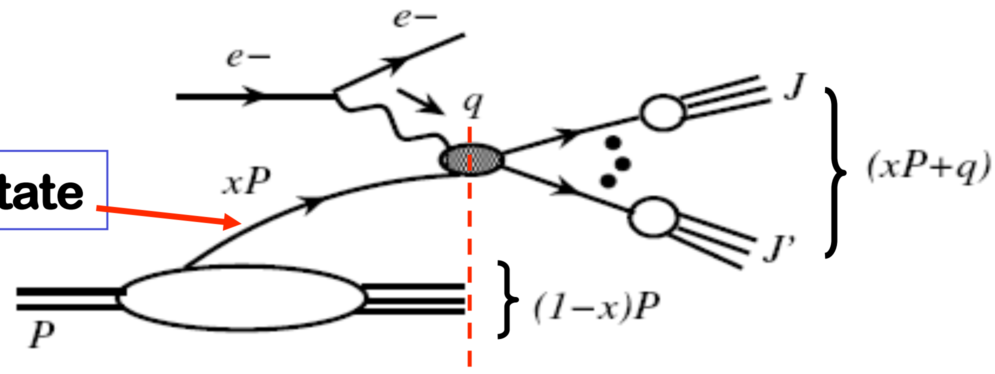
❑ Structure functions:

$$F_T, \ F_L \text{ or } F_1, \ F_2 \quad (F_3 \text{ for parity violating interaction})$$

Picture of pQCD Factorization for DIS

- ❑ **Time evolution:**

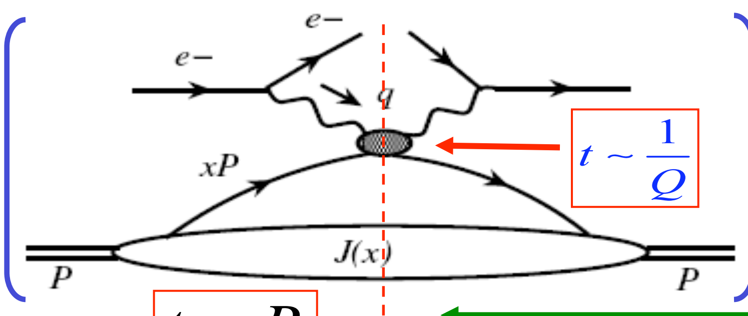
Long-lived parton state



Time: "Past" **Now** "Future"

- ❑ **Unitarity – summing over all hard jets:**

$$\sigma_{\rm tot}^{\rm DIS} \propto \mathrm{Im}$$



$t \sim \frac{1}{Q}$

+ O(1/Q)

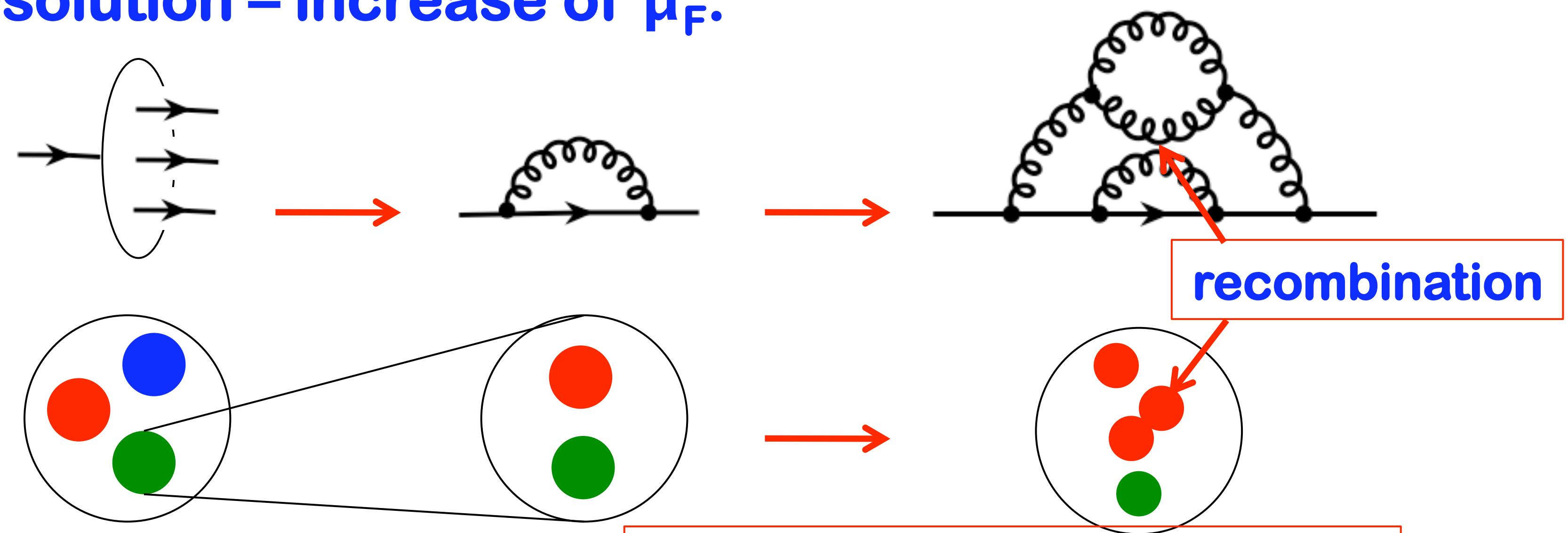
$t \sim R$

Not IR safe

$$= \sum_f \int dx\, \phi_f(x,\mu_F^2)\, \hat{\sigma}_f(x_B/x, Q^2/\mu_F^2, \alpha_s(\mu)) + \mathcal{O}(1/Q)$$

Factorization Scale Dependence – "Evolution"

- Resolution – increase of μ_F:

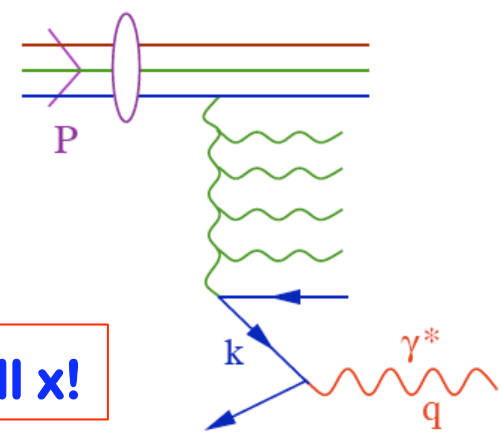


More partons at a smaller $1/\mu_F$

- DGLAP evolution:

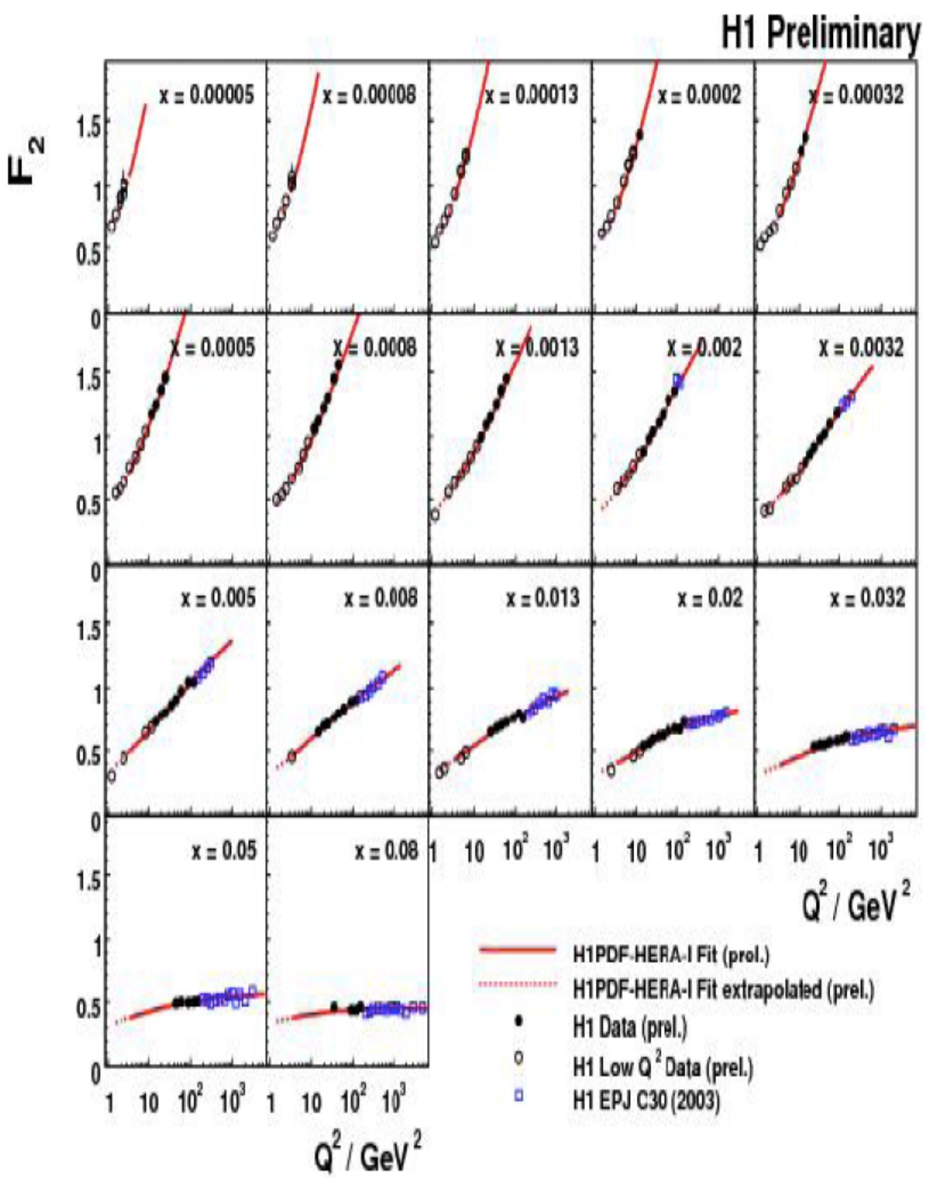
$$\frac{\partial \phi_g(x,\mu^2)}{\partial \ln(\mu_F^2)} = P_{gg}(x) \otimes \phi_g(x,\mu^2) + \ldots$$

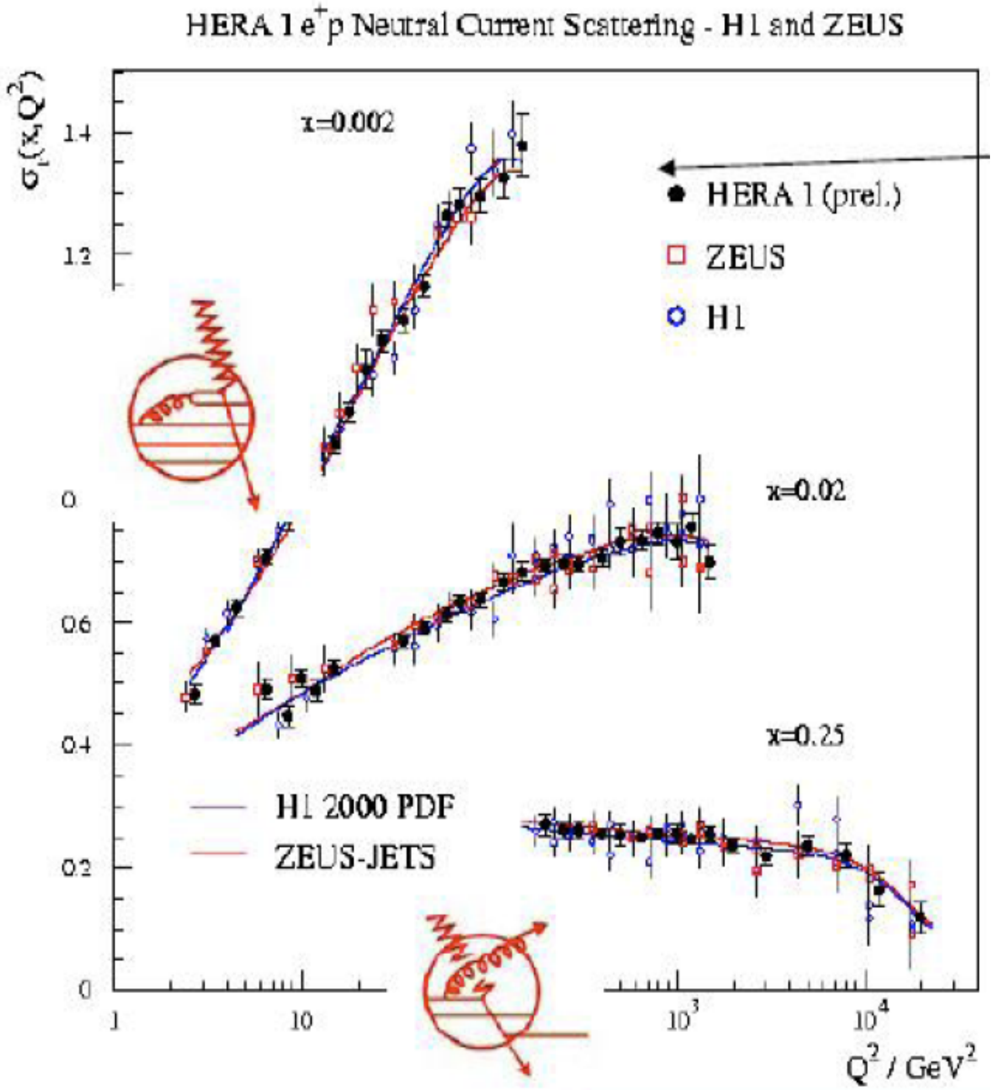
$$P_{gg}(x) \propto \frac{1}{x}$$



Large number gluons (sea quarks) at small x!

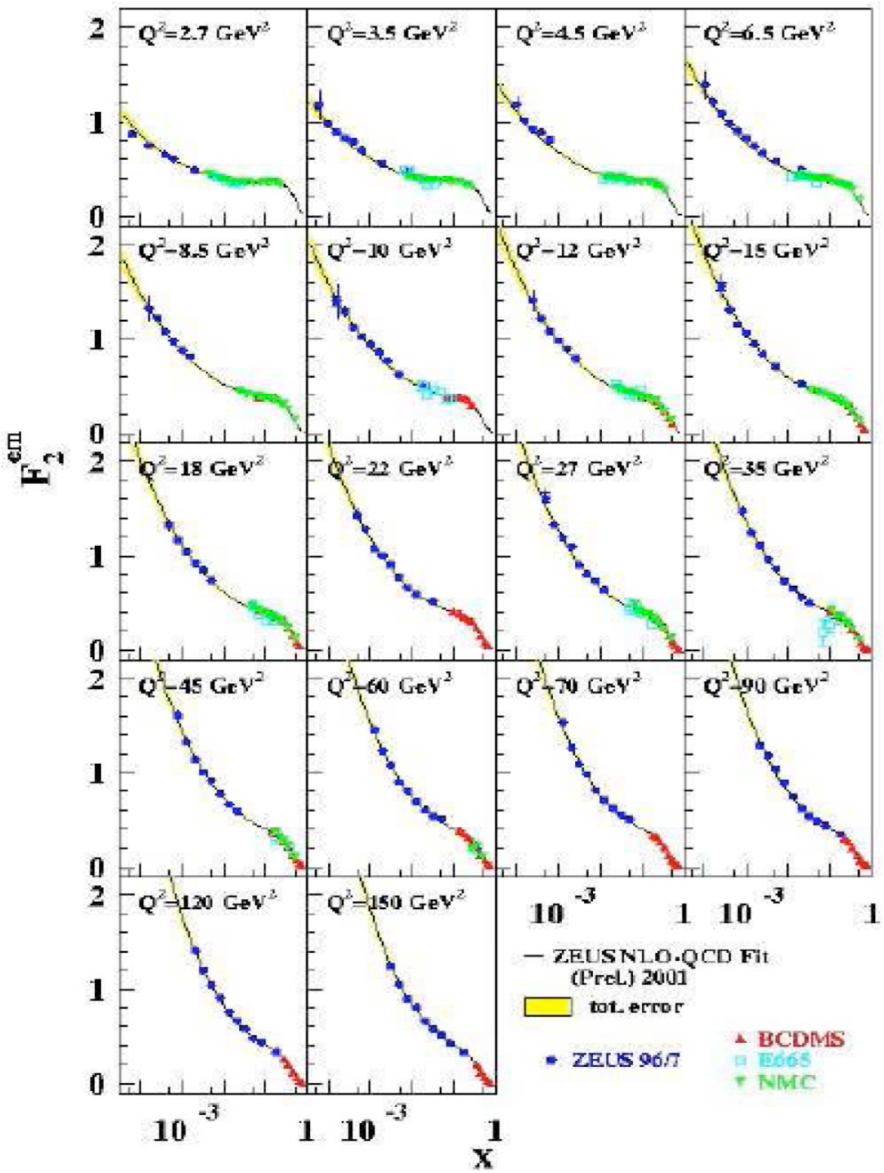
Structure Functions as a Function of Q^2

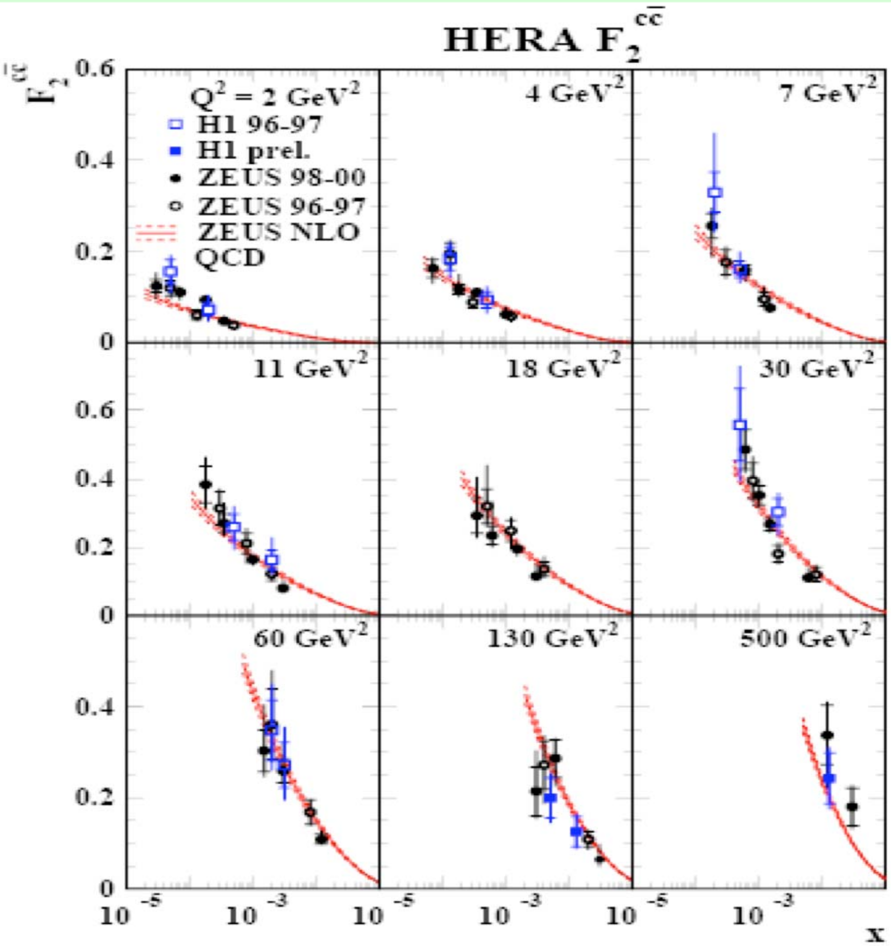




DGLAP works!

Structure Functions as a Function of x_B





✧ **At Q~1.5 GeV, F_2 still grows**

✧ **F_2(charm) or gluon also grows**

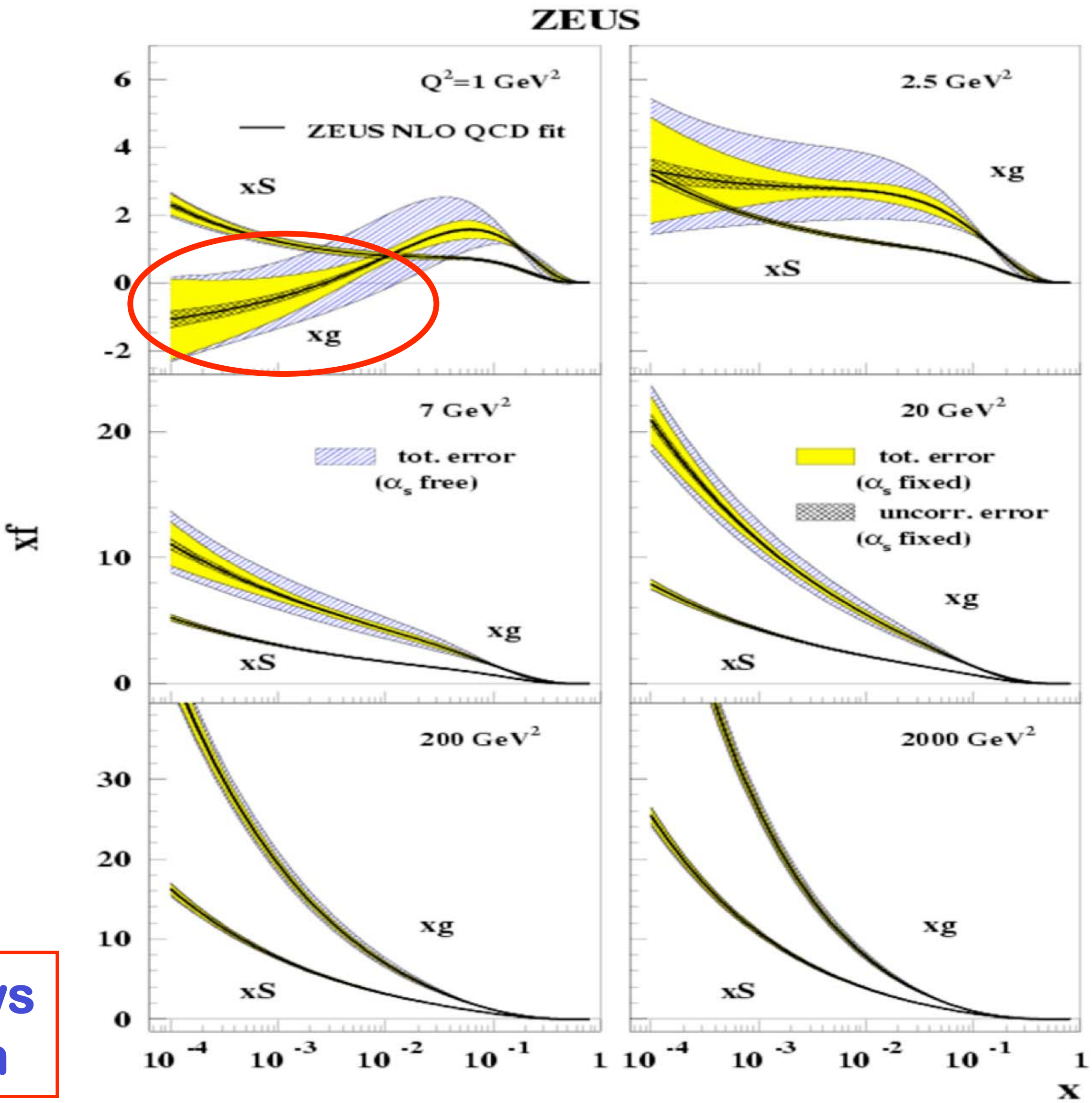
Negative gluon distribution at low Q?

- NLO global fitting based on leading twist DGLAP evolution leads to negative gluon distribution
- MRST, CTEQ PDF's have the same features

Does it mean that we have no gluon for $x < 10^{-3}$ at 1 GeV?

No!

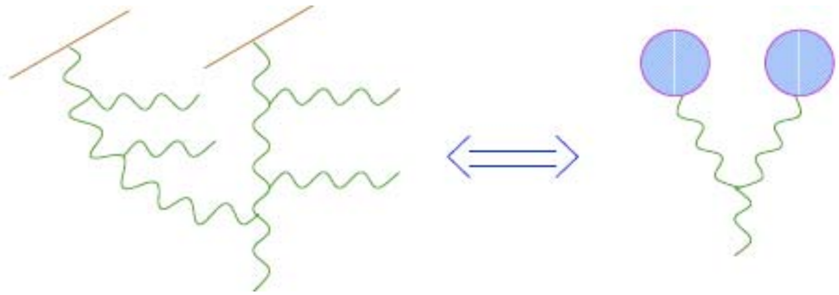
Power corrections slows down small-x evolution



Modified Evolution – Power Corrections

❑ Parton recombination:

Gribov, Levin and Ryskin, 83



❑ Modified evolution:

Mueller, Qiu, 86

$$\frac{\partial \phi_g(x,\mu^2)}{\partial \ln(\mu^2)} = P_{gg}(x) \otimes \phi_g(x,\mu^2) - \frac{C}{Q^2 R^2}\mathcal{P}_{ggg}(x) \otimes [\phi_g(x,\mu^2)]^2 + \ldots$$

Only valid when the 2nd term is relatively small

Slow down the evolution
Prevent the gluon density to become negative

❑ Power corrections:

$$F(x_B,Q^2) = \sum_f c_f^{(2)}(x_B/x, Q^2/\mu^2) \otimes \phi_f(x,\mu^2) + \frac{1}{Q^2} c_f^{(4)}(x_B/x, Q^2/\mu_F^2) \otimes \phi_f^{(4)}(x,\mu^2) + \ldots$$

Parton Saturation

- **Saturation: Radiation = Recombination** *Mueller, Qiu, 86*

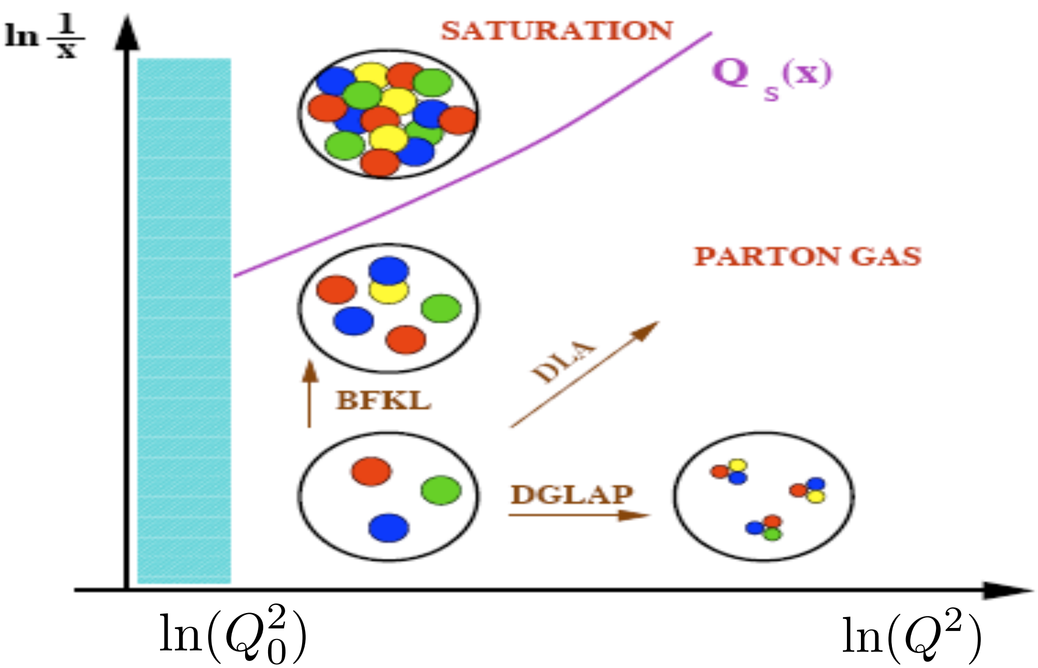
Estimate: $1 \approx \frac{8\pi\alpha_s}{3Q^2R^2}\, xG(x,Q^2)$

McLerran, Venugopalan, 94
And …

- **Saturation scale:** $Q_s^2(x) \simeq \alpha_s \frac{xG(x,Q_s^2)}{\pi R^2} \sim \frac{1}{x^\lambda}$

$Q^2 \gg Q_s^2(x)$: Dilute regime (rapid growth: BFKL, DGLAP)

$Q^2 \lesssim Q_s^2(x)$: Saturation: $n \sim 1/\alpha_s$ (large but constant)



Proton is dilute enough

Use nuclear target!

✧ **How to approach the saturation region?**

✧ **How to treat the saturation in QCD?**

Hard probe and its probing size

- Hard probe – process with a large momentum transfer:

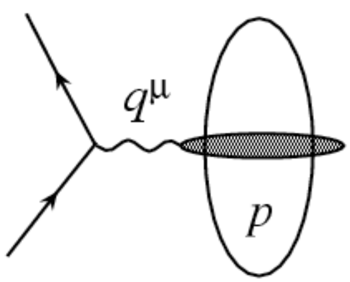
$$q^\mu \quad \text{with} \quad Q \equiv \sqrt{|q^2|} \gg \Lambda_{\rm QCD}$$

- Size of a hard probe is very localized and much smaller than a typical hadron at rest:

$$\frac{1}{Q} \ll 2R \sim \text{fm}$$

- But, it might be larger than a Lorentz contracted hadron:

$$\frac{1}{Q} \sim \frac{1}{xp} \gg 2R\left(\frac{m}{p}\right) \quad \text{or equivalently} \quad x \ll x_c \equiv \frac{1}{2mR} \sim 0.1$$



If an active parton *x* is small enough
the hard probe could cover several nucleons
in a Lorentz contracted large nucleus!

Coherence length in different frames

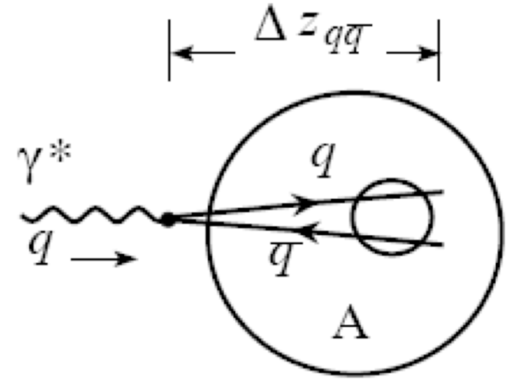
- **In target rest frame:**

  – Lifetime of the $q\bar{q}$ state:

$$\Delta E_{q\bar{q}} \sim \nu - E_{q\bar{q}} \sim \frac{Q^2}{2\nu}\left[1 + \mathcal{O}\left(\frac{m_{q\bar{q}}^2}{Q^2}\right)\right]$$

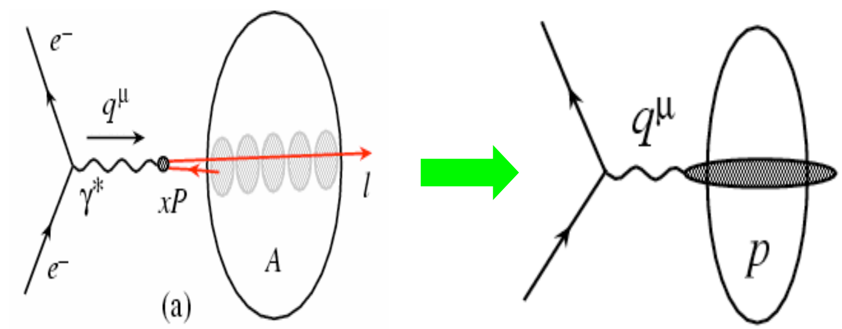
$$\Delta z_{q\bar{q}} \sim \frac{1}{\Delta E_{q\bar{q}}} \sim \frac{2\nu}{Q^2} = \frac{1}{m x_B}$$

  – $\Delta z_{q\bar{q}} \gg 2$ fm, inter-nuclear distance, if $x_B \ll 0.1$



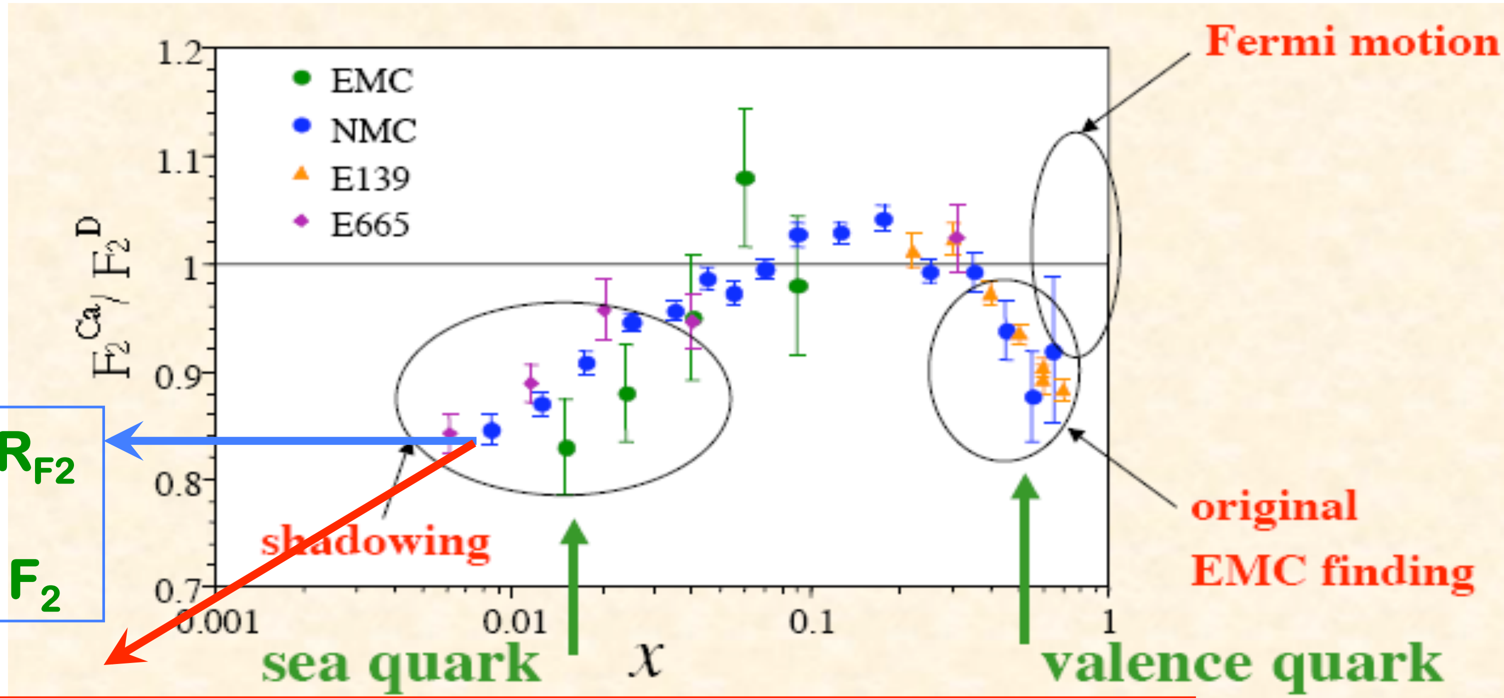
- **If $x_B \ll 0.1$, the q-qbar state of the virtual photon can interact with whole hadron/nucleus coherently.**

The conclusion is frame independent



What have we learned from eA collisions?

- **EMC effect, Shadowing and Saturation:**



Saturation in R_{F2}
$\neq$
Saturation in F_2

Saturation in $F_2(A) = R_{F2}$ decreases until saturation in $F_2(D)$

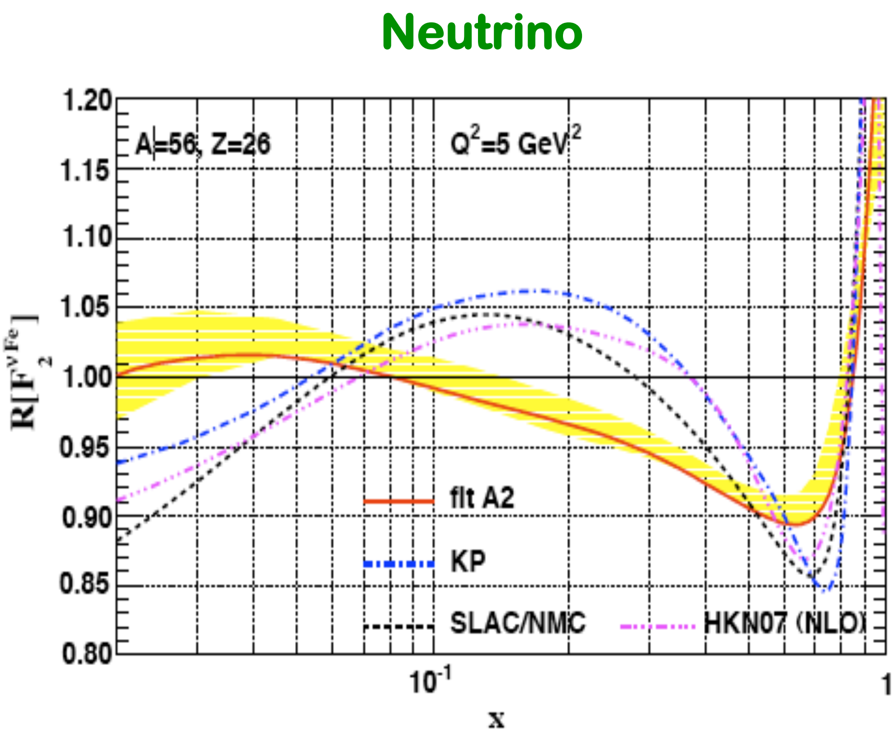
- **EIC – R_{F2} as a function of x_B at a fixed Q^2 for various A**

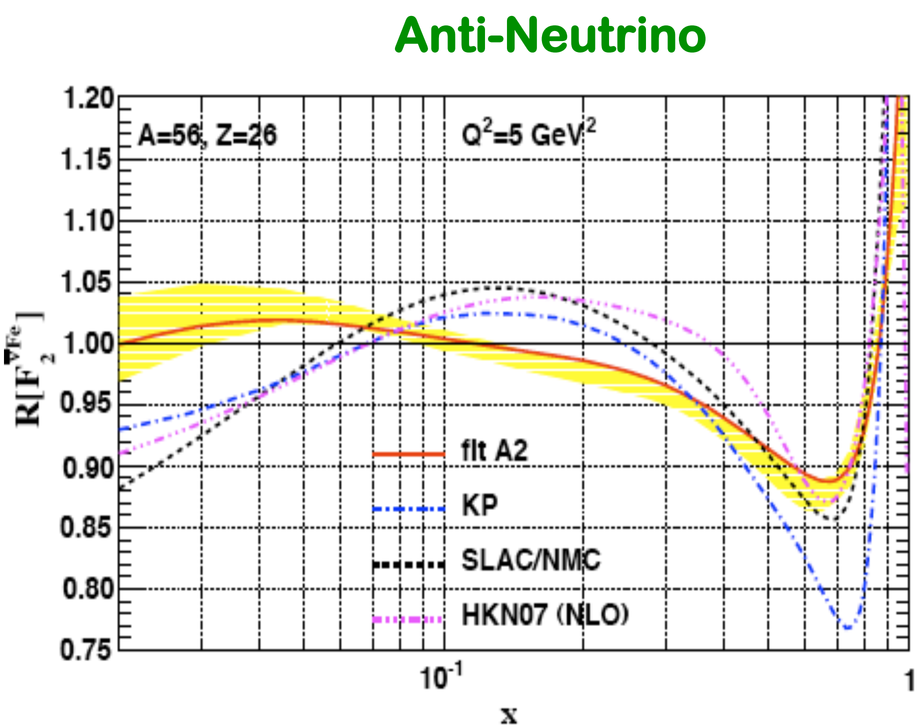
Need x_B as small as 10^{-3} at Q^2=2GeV^2 to probe the saturation

Surprise from the neutrino-A experiments

Schienbein, et al. PRD 2008

- CTEQ global fits for nuclear PDFs:





CTEQ fits prefer no shadowing for R_{F2}, and and a shifted "antishadowing" region

Q: Universality of nPDFs? A larger power correction?

Power Corrections to inclusive DIS

❑ Operator Product Expansion (OPE):

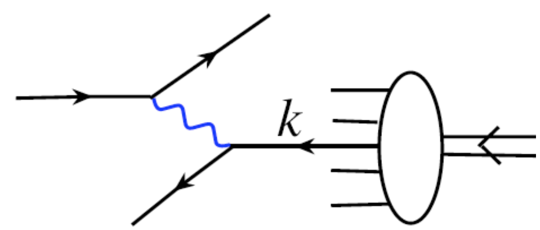
Should work for inclusive DIS – pQCD collinear factorization

$$\sigma^{h}_{phys} = \hat{\sigma}^{i}_{2} \otimes [1 + C^{(1,2)}\alpha_s + C^{(2,2)}\alpha_s^2 + \ldots] \otimes T_2^{i/h}(x)$$
$$+ \frac{\hat{\sigma}^{i}_{4}}{Q^2} \otimes [1 + C^{(1,4)}\alpha_s + C^{(2,4)}\alpha_s^2 + \ldots] \otimes T_4^{i/h}(x)$$
$$+ \frac{\hat{\sigma}^{i}_{6}}{Q^4} \otimes [1 + C^{(1,6)}\alpha_s + C^{(2,6)}\alpha_s^2 + \ldots] \otimes T_6^{i/h}(x)$$
$$+ \ldots$$

Leading twist

Power corrections

❑ Breakdown of collinear factorization:



when the active parton momentum:

$$k^{+} = xp = Q \sim k_T$$

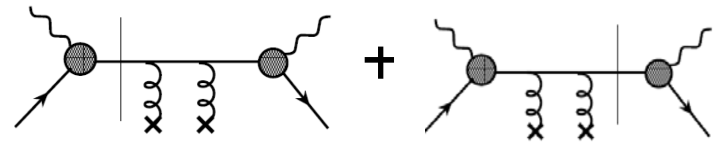
Leading tree-level power correction

- **LO contribution:**

Qiu, Vitev, 2004

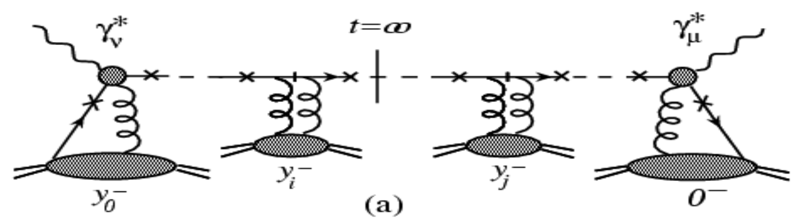
$$\Rightarrow \quad \delta(x-x_B)$$

- **NLO contribution:**



$$\Rightarrow \quad \frac{g^2}{Q^2}\left(\frac{1}{2N_c}\right)\underbrace{\left[2\pi^2\tilde{F}^2(0)\right]}_{\int \frac{dy_2^-\,dy_1^-}{(2\pi)^2}\left[F^{+\alpha}(y_2^-)F_{\alpha}^{\ +}(y_1^-)\right]\theta(y_2^-)}\, x_B \underbrace{\lim_{x_1\to x}\left[\frac{1}{x-x_1}\delta(x_1-x_B)+\frac{1}{x_1-x}\delta(x-x_B)\right]}_{x_B\left[-\frac{d}{dx}\delta(x-x_B)\right]}$$

- **Nth order contribution:**



$$\Rightarrow$$

$$\left[\frac{g^2}{Q^2}\left(\frac{1}{2N_c}\right)\left[2\pi^2\tilde{F}^2(0)\right]\right]^N \underbrace{x_B^N \lim_{x_i\to x}\sum_{m=0}^{N}\delta(x_m-x_B)\left[\prod_{i=1}^{m}\left(\frac{1}{x_{i-1}-x_m}\right)\right]\left[\prod_{j=1}^{N-m}\left(\frac{1}{x_{m+j}-x_m}\right)\right]}_{x_B^N\left[(-1)^N\frac{1}{N!}\frac{d^N}{dx^N}\delta(x-x_B)\right]}$$

$\boxed{\text{Infrared safe!}}$

Contributions to DIS structure functions

Qiu and Vitev, PRL (2004)

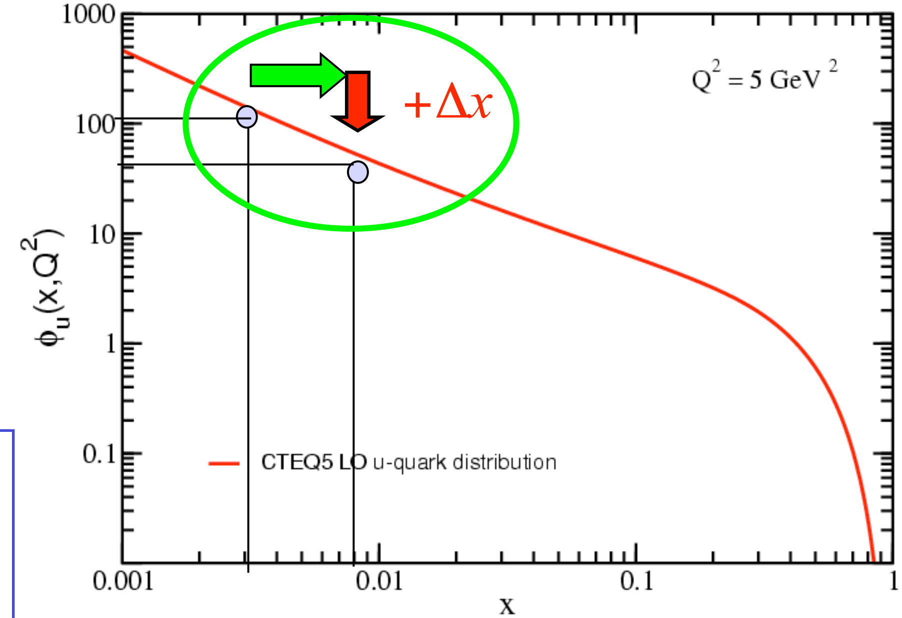
- **Transverse structure function:**

$$F_T(x_B,Q^2) = \sum_{n=0}^{N} \frac{1}{n!}\left[\frac{\xi^2}{Q^2}\left(A^{1/3}-1\right)\right]^n x_B^n \frac{d^n}{dx_B^n} F_T^{(0)}(x_B,Q^2)$$

$$\approx F_T^{(0)}(x_B(1+\Delta),Q^2)$$

$$\Delta \equiv \frac{\xi^2}{Q^2}\left(A^{1/3}-1\right)$$

$$\xi^2 = \frac{3\pi\alpha_s}{8R^2}\langle F^{+\alpha}F_\alpha{}^{+}\rangle$$

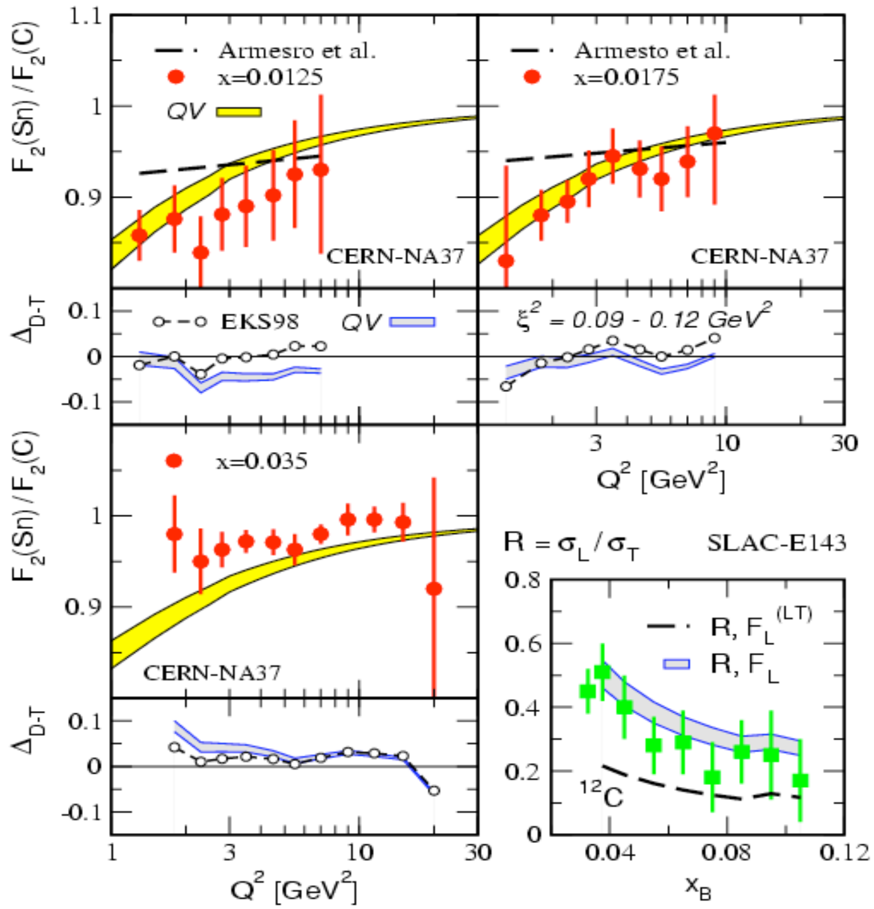


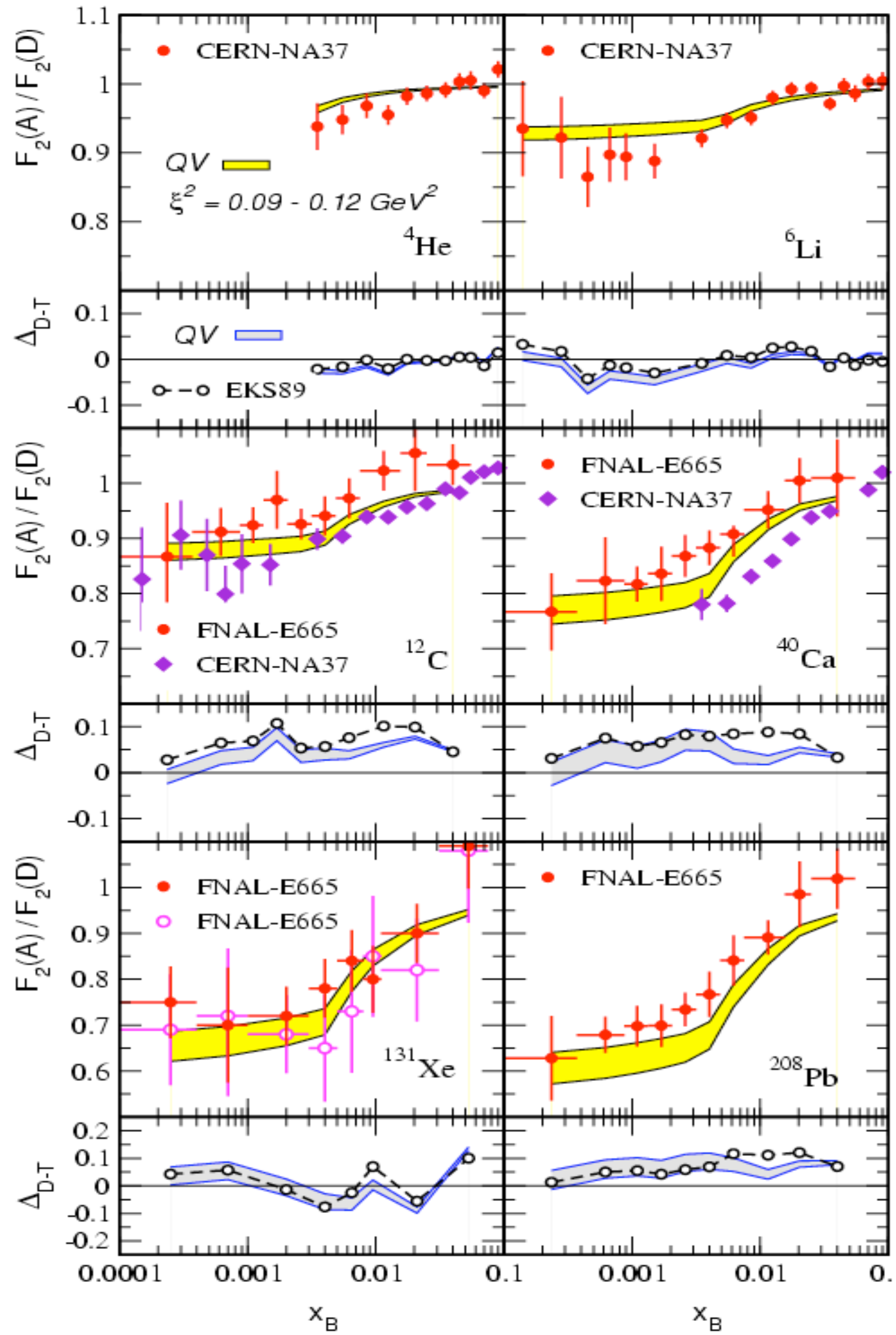
Single parameter for the power correction, and is proportional to **the same characteristic scale**

- **Similar result for longitudinal structure function**

Neglect LT shadowing upper limit of ξ^2

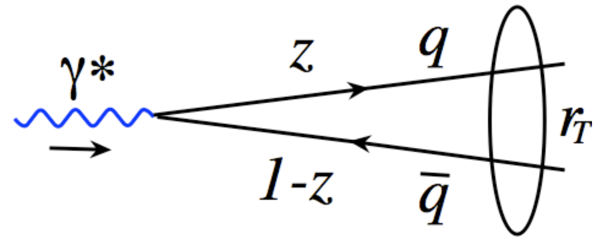
$\xi^2 \sim$ 0.09 - 0.12 GeV^2





Golec-Biernat and Wustoff Model

❑ In target rest frame:



$$\sigma_{T,L}^{\gamma^* p} = \int d^2 r_T \int dz \left|\psi_{T,L}(r_T, z, Q^2)\right|^2 \sigma_{q\bar{q}p}(r_T, x)$$

$$\sigma_{q\bar{q}p}(r_T, x) = \sigma_0 \left[1 - \exp(-r_T^2\, Q_s^2(x))\right]$$

❑ Saturation scale:

$$Q_s^2(x) \equiv Q_0^2 \left(\frac{x_0}{x}\right)^\lambda$$

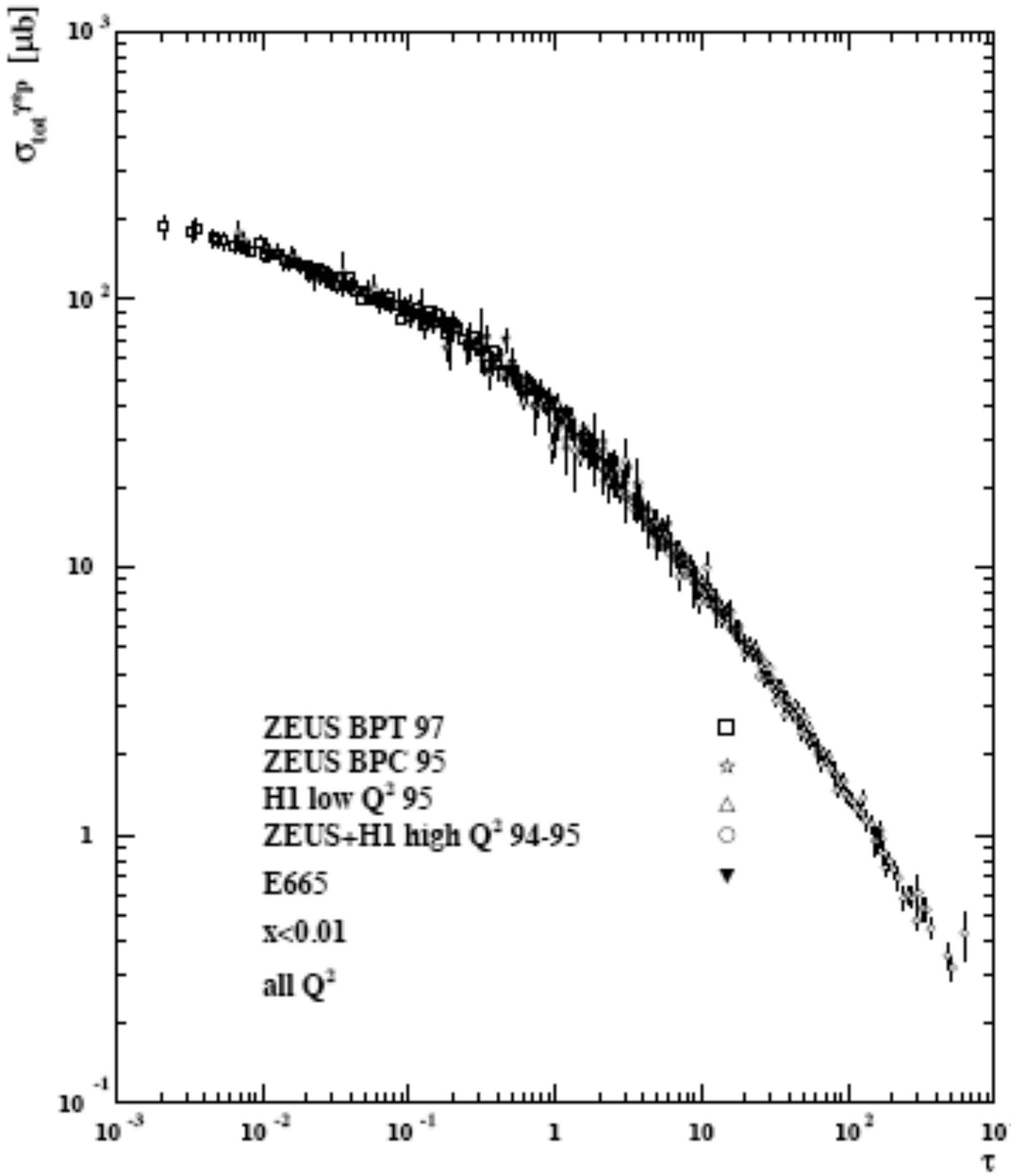
Fix all four parameters by fitting all HERA data with x<0.01 and all Q

$$Q_0 = 1\,\mathrm{GeV};\;\; \lambda = 0.3;\;\; x_0 = 3\cdot 10^{-4};\;\; \sigma_0 = 23\,\mathrm{mb}$$

❑ Prediction - geometric scaling:

$$\sigma_{T,L}^{\gamma^* p} = f_{T,L}(Q^2/Q_s^2(x))$$

Geometric Scaling in HERA data



$$\tau \equiv \frac{Q^2}{Q_s^2(x)}$$

$$0.045 \leq Q^2 \leq 450\mathrm{GeV}^2$$

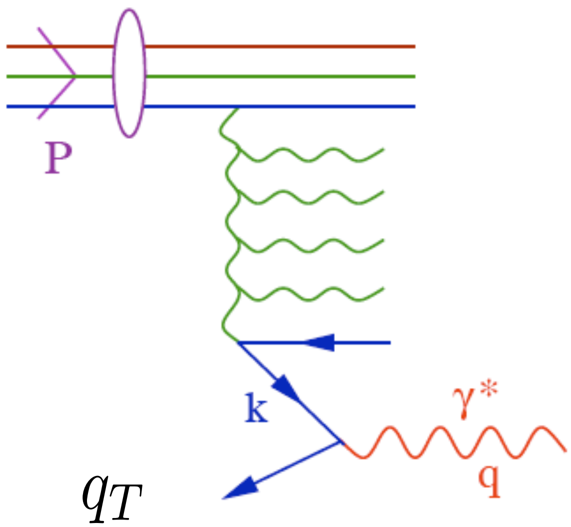
Why the model is so successful?

- ❑ **Dipole cross section:**

$$\sigma_{q\bar{q}p}(r_T, x) = \sigma_0 \left[1 - \exp(-r_T^2\, Q_s^2(x))\right]$$

- ✧ Controlled by the transverse size of the qqbar pair
- ✧ The size is characterized by the $Q_s^2(x)$

- ❑ **In center of mass frame – NLO pQCD fits the data too:**



The geometric scaling indicates that the cross section is mainly determined by the qqbar states with the transverse momentum characterized by

$$Q_s^2(x) \propto \frac{1}{x^{0.3}}$$

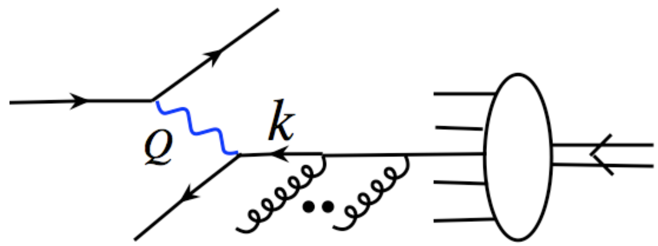
Q: Can we verify this in pQCD collinear factorization approach?

Semi-inclusive DIS in ep Collisions

- **Collision energies:**

$$S_{\gamma^*-A} = (q+p)^2 \approx Q^2\left[\frac{1-x_B}{x_B}\right] \sim \frac{Q^2}{x_B}$$

- **Single hadron production at p_T:**



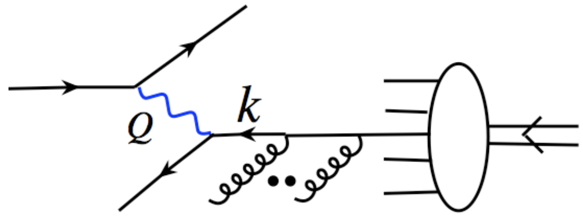
  - ✧ **Hard scale: Q assures a hard collision and pQCD calculation**
  - ✧ **Soft Scale: p_T probes parton's transverse momentum at the collision point**

- **Parton's transverse momentum at the hard collision:**

  - ✧ **is not equal to 1/fm – typical scale in hadron wave function**
  - ✧ **Gluon shower from both initial state and final-state partons, and soft interaction between them can all change the p_T**

Gluon Shower when q_T is small

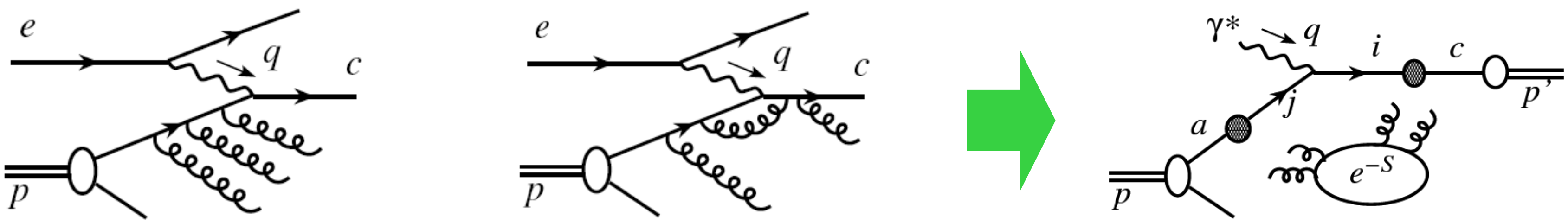
- When q_T is small, fixed order calculation diverges:



LO: $\frac{\alpha_s}{q_T^2}\left[a + b\log(Q^2/q_T^2)\right] \;\rightarrow \infty$ as $q_T^2 \rightarrow 0$

initial-state and final-state soft gluon radiations generate large logarithms: $\frac{1}{q_T^2}\alpha_s^n \log^{2n-1}(Q^2/q_T^2)$

- QCD resummation:

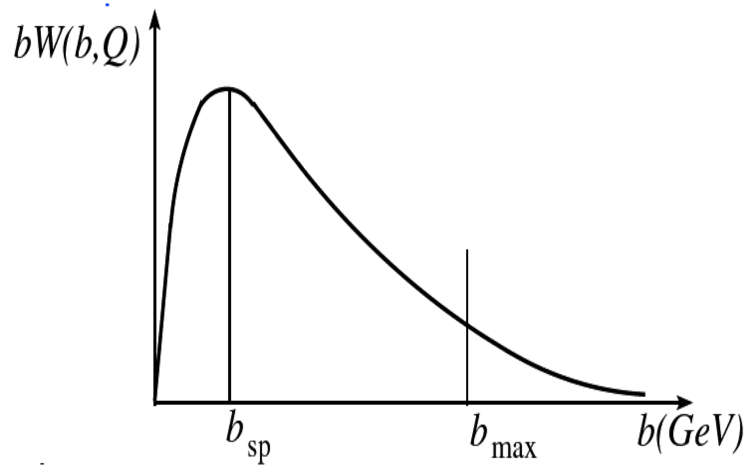


Calculation in the b-space

❑ Resummed x-section:

$$\frac{d\sigma^{(\mathrm{resum})}_{A\to h}}{dx_B\, dQ^2\, dz\, dq_T^2} \propto \int \frac{d^2 b}{(2\pi)^2}\, e^{i\vec{q}_T\cdot\vec{b}}\, W(b,x,z,Q)$$

$$W = \begin{cases} W^{\mathrm{pert}}(b,x,z,Q) & b \le b_{max} \\ W^{\mathrm{pert}}(b_{max},x,z,Q)\, F^{NP}(b,Q;b_{max}) & b > b_{max} \end{cases}$$



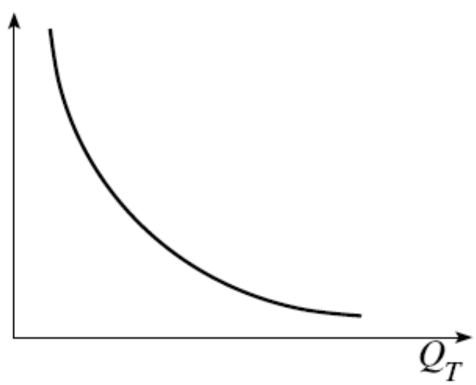
$$W^{\mathrm{pert}}(b,x,z,Q) = \sum_j e_j^2 \left[f_{a/A} \otimes C^{in}_{a\to j}\right] \left[C^{out}_{j\to c} \otimes D_{b\to h}\right] \times e^{-S(b,Q)}$$

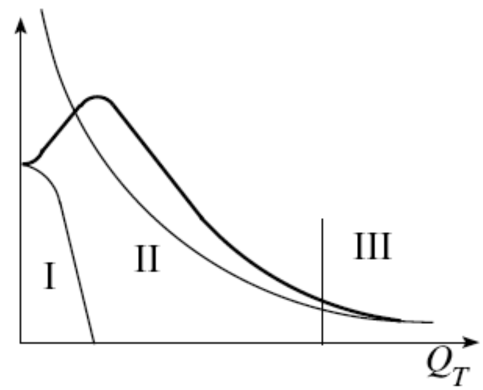
❑ Features:

- Sudakov form factor $\to$ $b_{sp} \propto (\frac{\Lambda_{QCD}}{Q})^\lambda, \lambda \sim 0.5$
- evolution of $f_{a/A}$ and $D_{c\to h}$ also moves b_{sp} smaller $\xi \Rightarrow \mu\frac{\partial}{\partial\mu} f_{a/A}(\xi) > 0 \Rightarrow$ lower b_{sp}

Resummed Q_T Distribution

❑ Remove the divergence:





❑ Features:

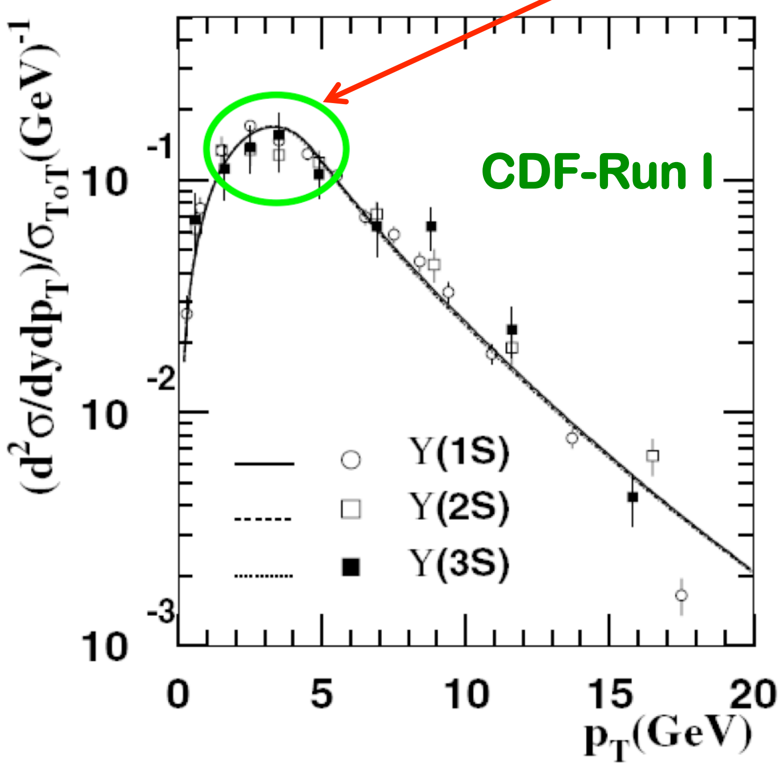
- (I): dominated by intrinsic k_T (Gaussian type)
- (II): pQCD soft-gluon resummation ($q_T \leq Q$)
- (III): pQCD fixed order calculation ($q_T \sim Q$)
- relative size of three regions depend on Q^2 and S
- large Q^2 and large $S \Rightarrow$ smaller region (I)
- smaller $Q^2 \rightarrow$ smaller logs $\rightarrow$ smaller region (II)

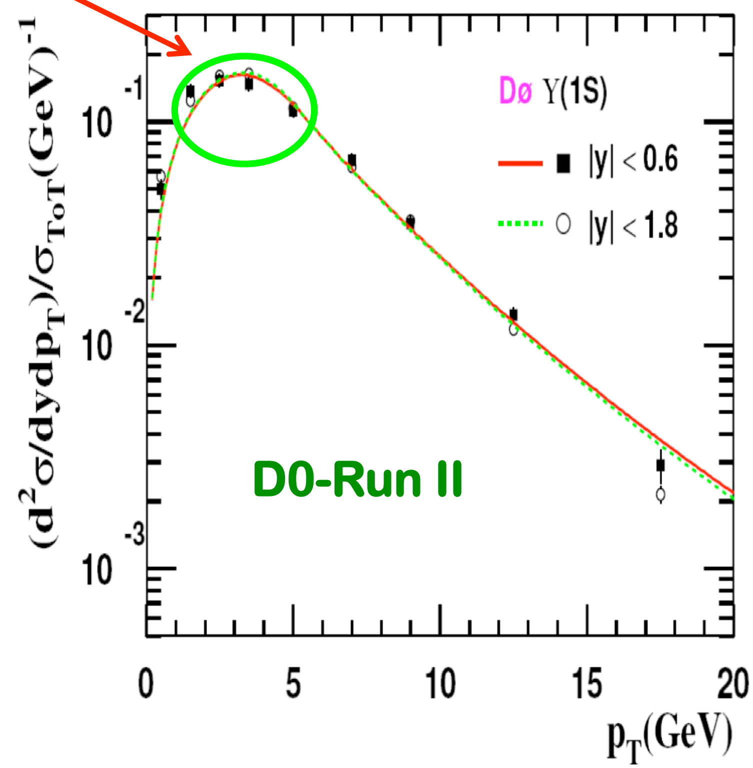
Works for heavy boson production

- Upsilon at Tevatron:

Dominated by perturbative small-b contribution in its Fourier conjugate space

Berger, Qiu, Wang, 2005





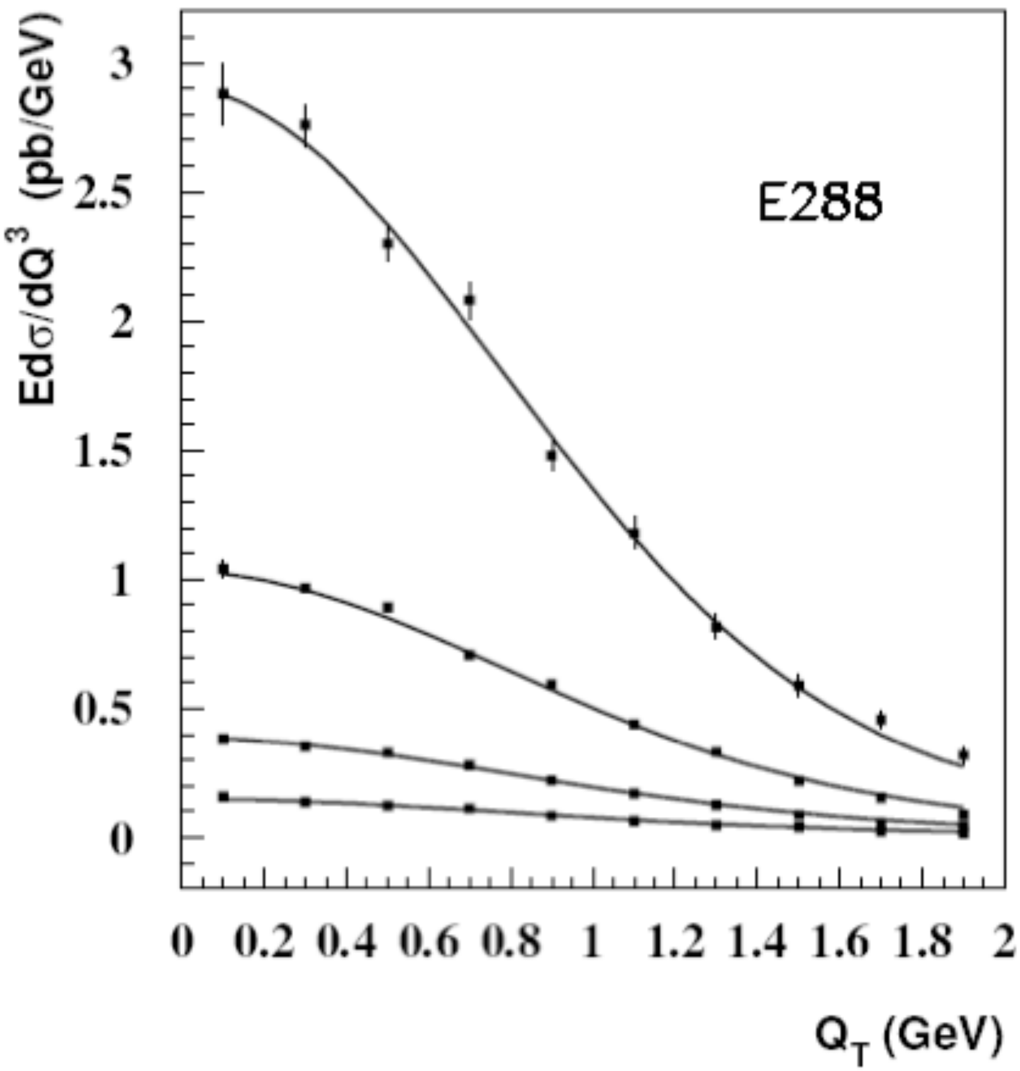
- Works better for W/Z, also work for Drell-Yan, …

Qiu, Zhang, 2001

Comparison with Fermilab Drell-Yan data

❑ **Low energy data:**

Qiu, Zhang, 2001

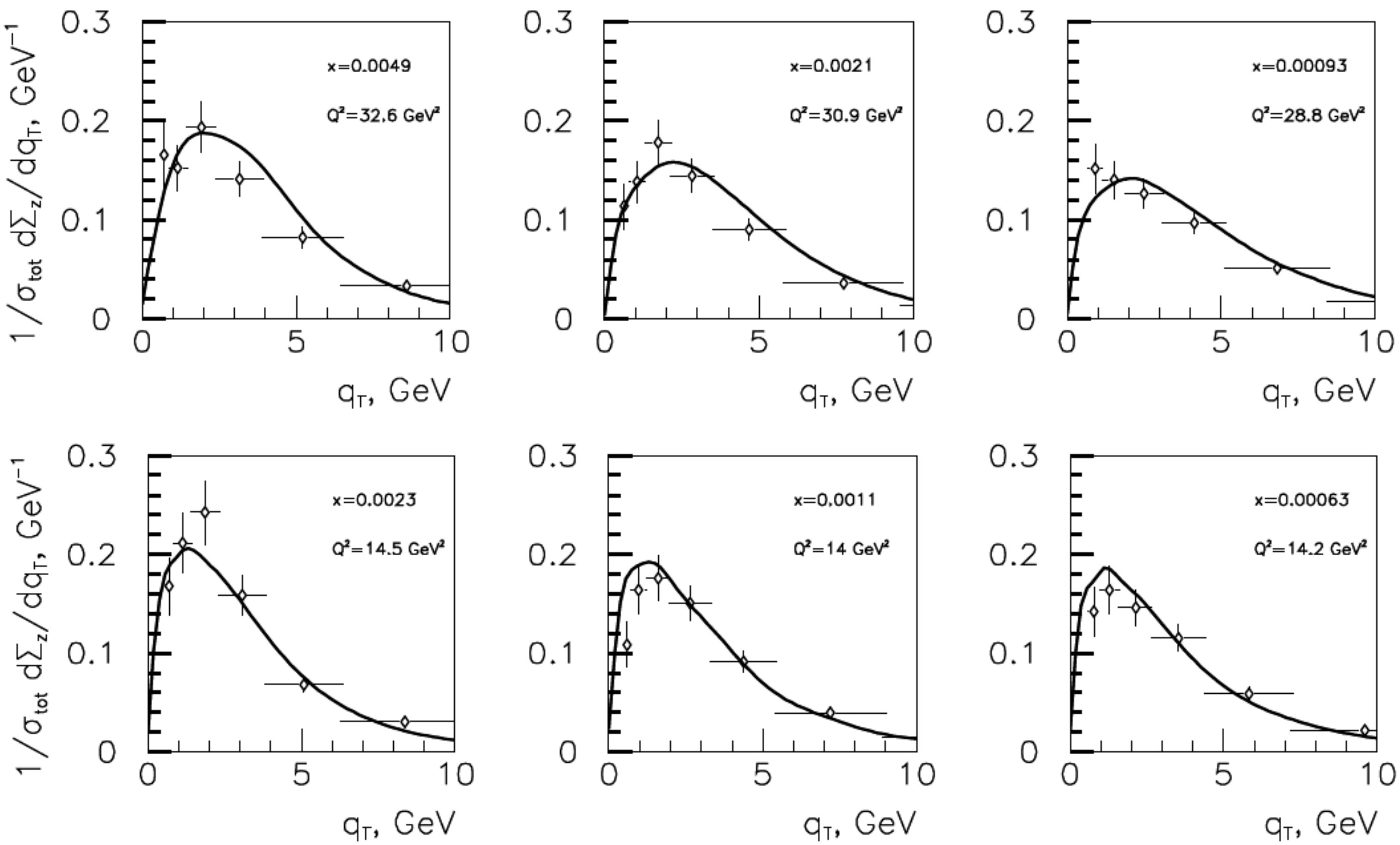


E288 data

P_{beam} = 400 GeV

Also work for HERA data

$$\sum_h \int dz\, z\, \frac{d\sigma_{A\to h}}{dx_B\, dQ^2\, dz\, dq_T^2}$$

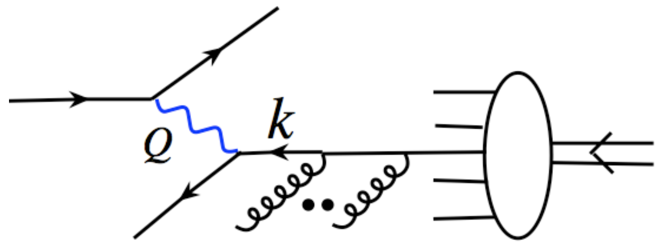


Nadolsky, et al, 1999, 2001

Perturbative "Saturation Scale"

Kang, Qiu, 2008

- The peak of the p_T distribution (b_{sp} in b-space):



Gluon shower is dominated the large phase space:

$$k_T^2 \propto \ln(S_{\gamma^*-p}/Q^2) \approx \ln(1/x_B) \sim 1/x^{1/3}$$

That is, pQCD factorization approach with resummation of coherent radiation can describe the same phenomenon

- But, the formalism does not apply when $Q^2 \sim 0.045\ \mathrm{GeV}^2$

That is, the target rest frame formalism is more suited for being extended into the region of saturation

How to describe the saturation in QCD?

- The high-density gluons are weakly coupled:

$$Q_s^2(x) \gg \Lambda_{\rm QCD}^2 \quad \Rightarrow \quad \alpha_s(Q_s^2) \ll 1$$

- With the large occupation number, their interaction is fully nonlinear:

$$\alpha_s\, n \sim 1$$

- Large occupation numbers combined the weak coupling

Strong classical "color" fields

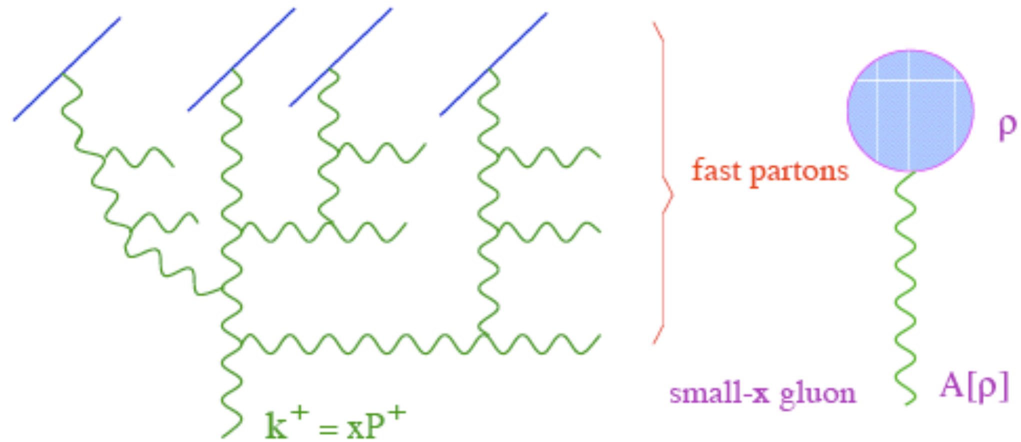
- Novel state: Color Glass Condenstate

A classical effective theory for the small-x gluons that is derived by integrating out the gluons of large x in pQCD

McLerran, Venugopalan (1994), improved by many peoples,
Iancu, Jalilian-Marian, Kovner, Leonidov, McLerran, Venugopalan , Weigert, ...

The Color Glass Condensate (CGC)

- The color source:



- Small-x gluons:

  Effectively given by the classical field $A[\rho]$ that is radiated By fast partons $(x' > x)$ having a color charge density ρ

- Large-x partons – charge density ρ :

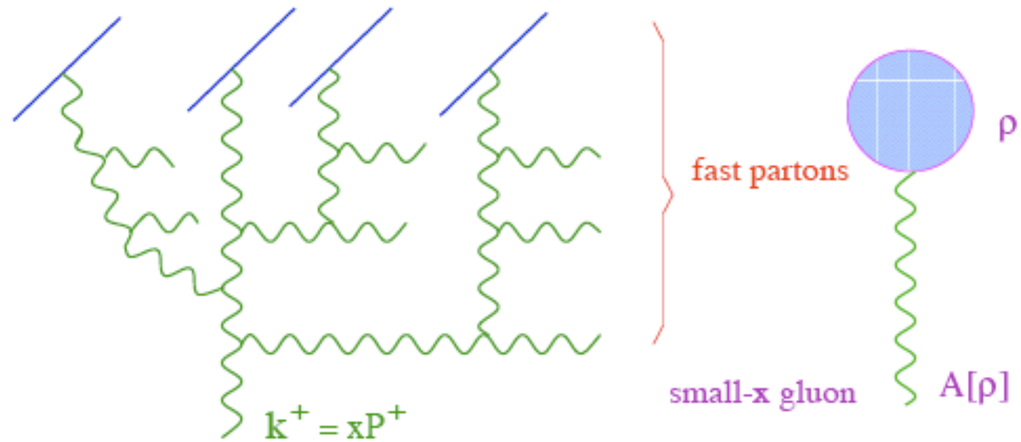
  Effectively frozen (time dilation) in some random configurations and have the probability charge distribution, $W_Y[\rho]$, $Y = \ln(1/x)$

- JIMWLK equation: $$\frac{\partial W_Y[\rho]}{\partial Y} = -H\left[\rho, \frac{\delta}{\delta\rho}\right] W_Y[\rho]$$

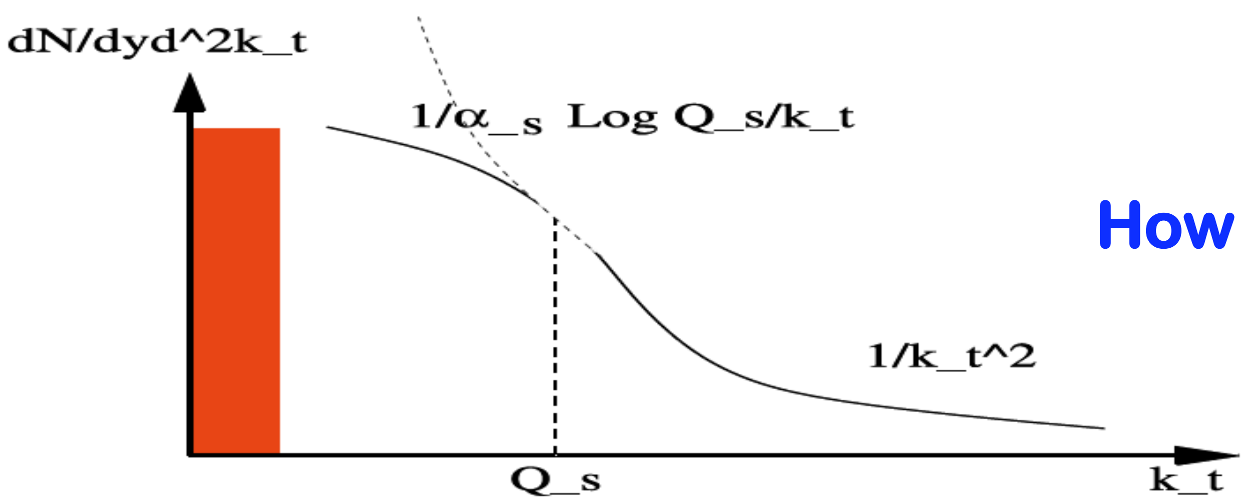
  Lower x, more phase space for fast partons and their radiation

Gluon Density of the CGC

- ❑ **The color source:**



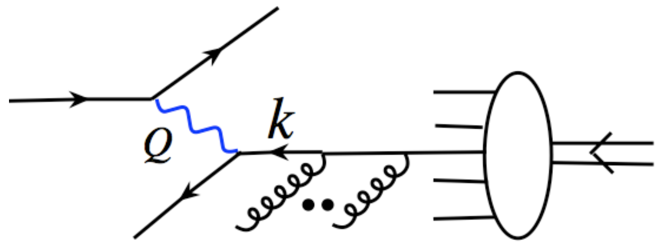
- ❑ **The gluon density:**
  - ✧ **Solve classical Yang-Mills equation of motion for the gluon field**
  - ✧ **Evaluate the $\langle A_T A_T \rangle$ to get the gluon density**



How to probe this?

Semi-inclusive DIS in eA Collisions

❑ **Parton's transverse momentum at the hard collision:**



Recall: for the "ep" case, gluon shower is mainly determined by

- ✧ the value exchange hard momentum Q, and
- ✧ the available phase space for the DGLAP evolution

If it is in the saturation regime, the gluon density does not evolve as fast as what DGLAP predicts, the therefore, the peak of p_T should shift toward a lower value

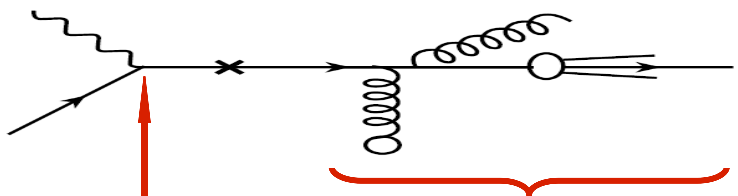
❑ **Mean transverse momentum square:**

$$\langle q_T^2 \rangle \equiv \int dq_T^2 \, q_T^2 \frac{d\sigma_{A\to h}}{dx_B dQ^2 dz dq_T^2} \Bigg/ \frac{d\sigma_{A\to h}}{dx_B dQ^2 dz}$$

Broadening in non-interacting nuclear matter

❑ **Induced radiation – energy lose:**

Guo & Wang PRL 2000, …
Wang & Wang, PRL 2002, …



Long-distance fragmentation

Short-distance hard scattering

❑ **Transverse momentum broadening – $A^{1/3}$ feature:**

$$\Delta\langle q_T^2\rangle \equiv \langle q_T^2\rangle^{hA} - \langle q_T^2\rangle^{hN} = \left(\frac{4\pi^2\alpha_s}{3}\right)\lambda^2 A^{1/3}$$

Guo, PRD 1998

– increases the effective "intrinsic k_T"

– reduces the phase space for soft-gluon shower

$\Rightarrow$ broadening the q_T distribution

Probe the Saturation

Kang, Qiu, 2008

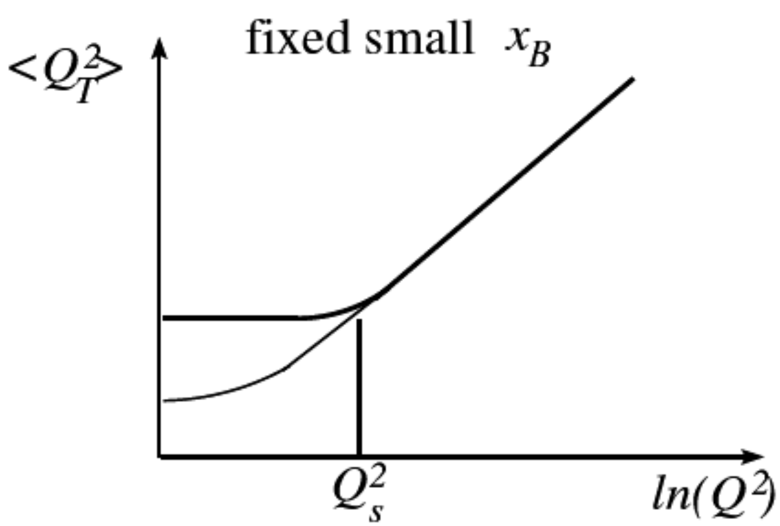
For a fixed A!

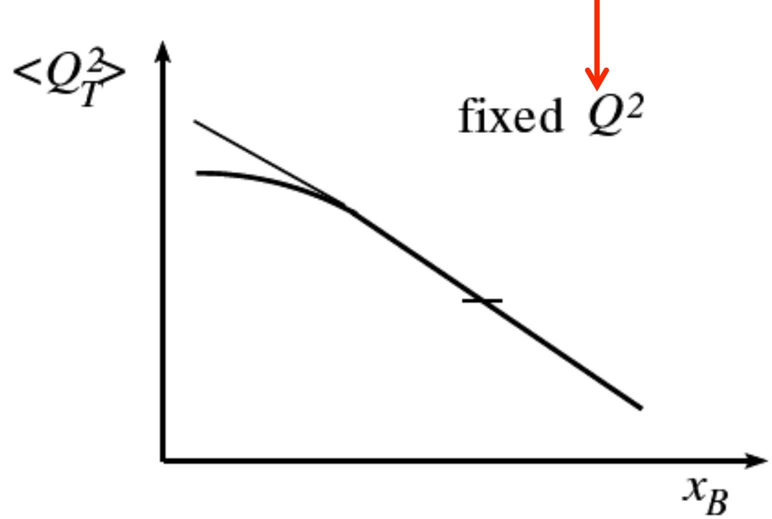
- **At small x_B (large $S_{\gamma^*\text{-}A}$), large phase space for shower**

Q_T-distribution could be calculable at low Q_T

- **Saturation stops the evolution:**

$$\Rightarrow \mu\frac{\partial}{\partial\mu} f_{a/A}(\xi) \to 0 \qquad \Rightarrow \text{increase } b_{sp}$$





Same measurement for a larger A!

Summary

- Many progresses made in our understanding of small-x physics – the partonic dynamics in a dense but weakly interacting medium of gluons.

  Many advances are made in connecting various evolution equations, such as the JIMWLK, Balitsky-Kovchegov, BFKL, DGLAP, …

- Semi-inclusive DIS in ep and eA provide many clean multiple scale observables

  – probe parton's transverse momentum off the CGC

- SIDIS and diffractive scattering, … in eA collisions

Provide many complementary observables to probe this novel low-x structure of matter

Thank you!