Single Transverse-Spin Asymmetry in Semi-inclusive D-meson Production

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Based on work done with Zhong-Bo Kang, PRD, 2008

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The Question

- How to probe the hadron structure beyond the PDFs?
  beyond the probability distributions?

$$\sigma(Q, s_T) = H_0 \otimes f_2 \otimes f_2 + (1/Q)\, H_1 \otimes f_2 \otimes f_3 + \mathcal{O}(1/Q^2)$$

Too large to compete!

Three-parton correlation

- Idea:

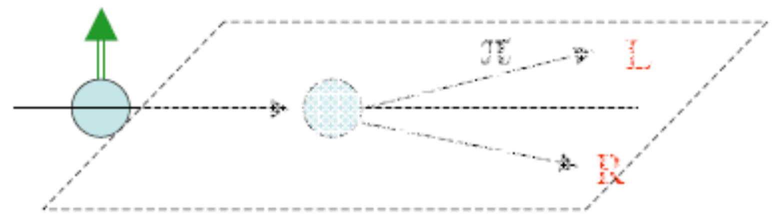
Take a difference of two cross sections,
whose leading power terms are canceled

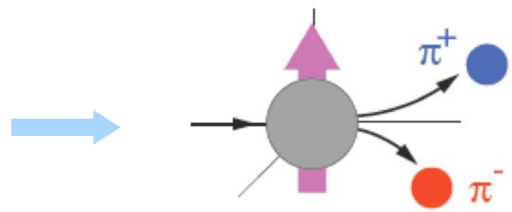
$$\Delta\sigma(Q, s_T) \equiv [\sigma(Q, s_T) - \sigma(Q, -s_T)]/2$$
$$= (1/Q) H_1(Q/\mu_F, \alpha_s) \otimes f_2(\mu_F) \otimes f_3(\mu_F) + \mathcal{O}(1/Q^2)$$

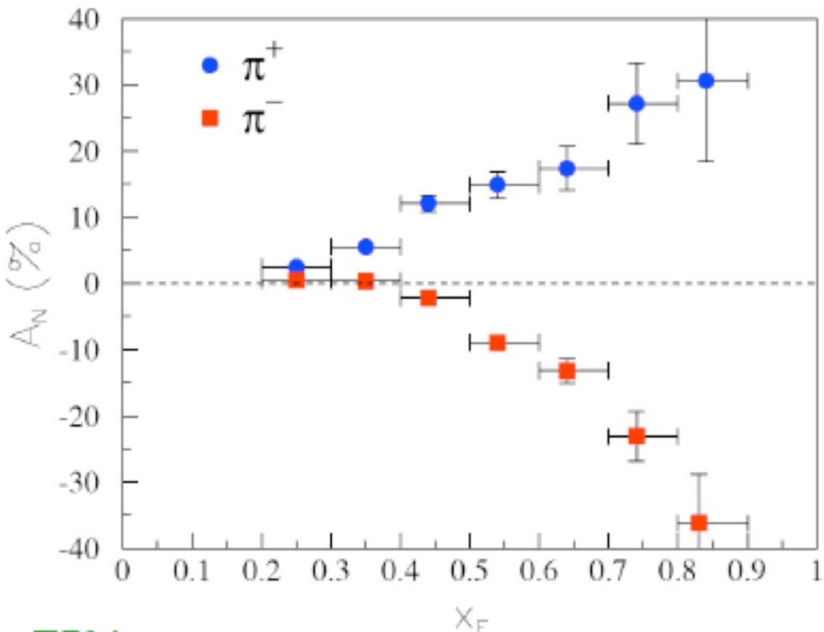
SSA in hadronic collisions

- **Hadronic** $p\uparrow + p \to \pi(l)X$ **:**

$$A_N = \frac{1}{P_{\text{beam}}} \frac{N^{\pi}_{\text{left}} - N^{\pi}_{\text{right}}}{N^{\pi}_{\text{left}} + N^{\pi}_{\text{right}}}$$







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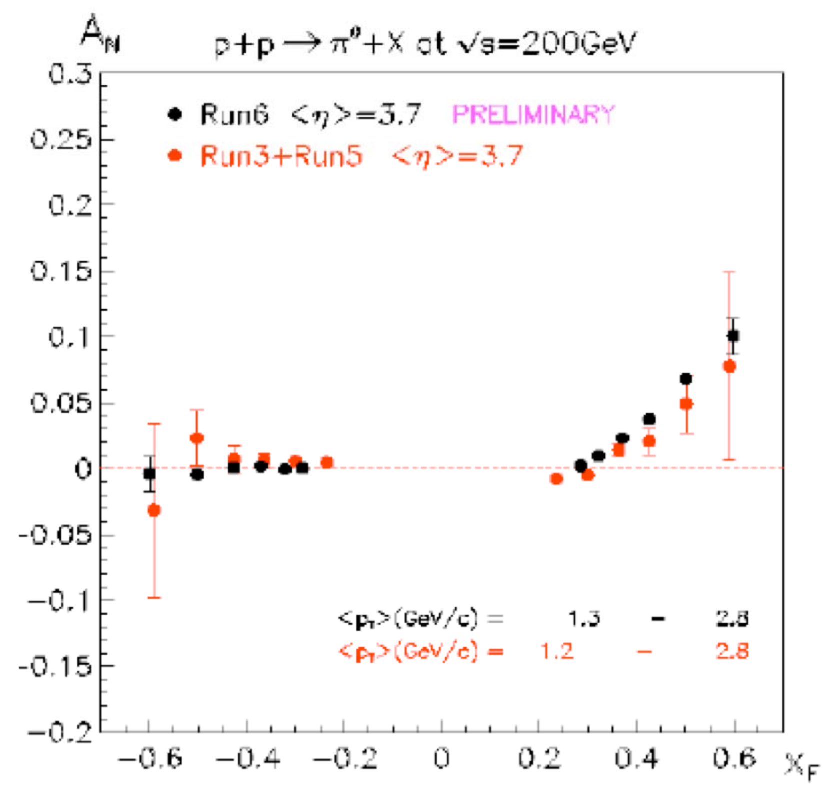


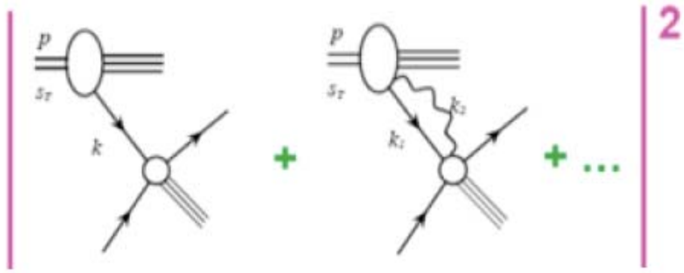
STAR (BRAHMS, too)

SSA in QCD Collinear Factorization

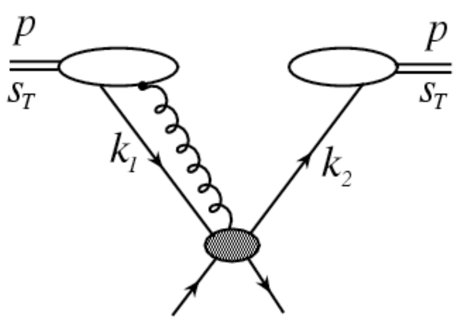
Efremov, Teryaev, 1982, Qiu, Sterman, 1991

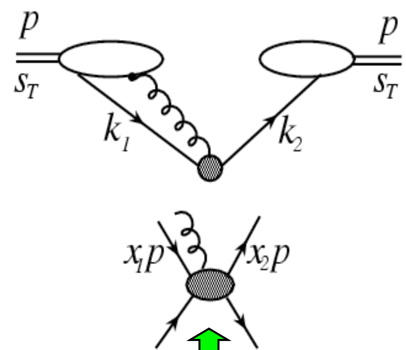
- **All scales >> Λ_{QCD}:**

$\sigma(s_T) \sim$



- **Factorization at twist-3:**





$T_{q,F}$

$$A_N \propto \frac{\int dk_T}{P_T}$$

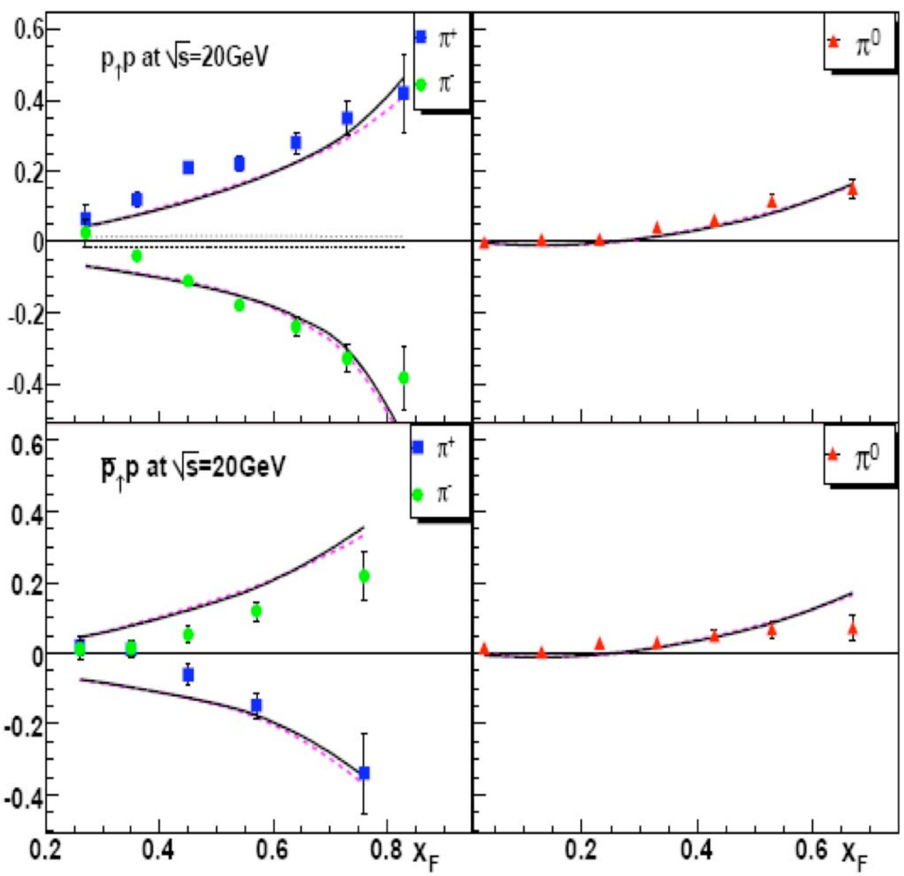
Normal twist-2 distributions

- **Twist-3 quark-gluon correlation:**

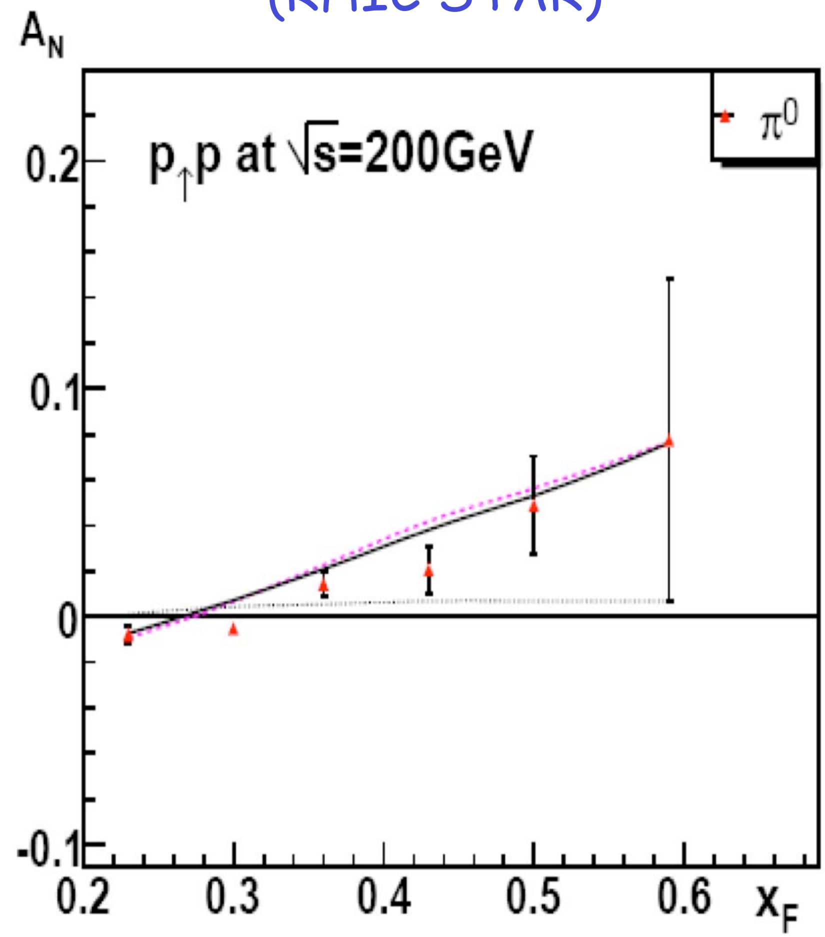
$$T_{q,F}(x,x,\mu_F) = \int \frac{dy_1^-}{2\pi} e^{ixP^+y_1^-} \langle P, s_T | \overline{\psi}_q(0) \frac{\gamma^+}{2} \left[\int dy_2^- \, \epsilon^{s_T \sigma n \bar{n}} F_\sigma{}^+(y_2^-) \right] \psi_q(y_1^-) | P, s_T \rangle$$

Initial Success of the Formalism

(FermiLab E704)



(RHIC STAR)



Kouvaris,Qiu,Vogelsang,Yuan, PRD, 2006

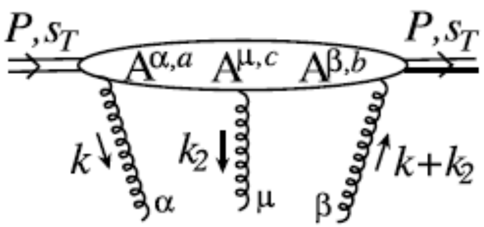
$T_{q,F}(x, x, \mu_F)$ **contribution only!**

Twist-3 tri-gluon correlations

Ji, 1992, Kang, Qiu 2008
Kang, Qiu, Vogelsang, Yuan, 2008

- Diagonal tri-gluon correlations:

$$T_G(x,x) = \int \frac{dy_1^- dy_2^-}{2\pi} e^{ixP^+y_1^-} \times \frac{1}{xP^+} \langle P, s_\perp | F^+{}_\alpha(0) \left[\epsilon^{s_\perp \sigma n \bar{n}} F_\sigma{}^+(y_2^-) \right] F^{\alpha +}(y_1^-) | P, s_\perp \rangle$$



- Two tri-gluon correlation functions – color contraction:

$$T_G^{(f)}(x,x) \propto i f^{ABC} F^A F^C F^B = F^A F^C (\mathcal{T}^C)^{AB} F^B$$

$$T_G^{(d)}(x,x) \propto d^{ABC} F^A F^C F^B = F^A F^C (\mathcal{D}^C)^{AB} F^B$$

Quark-gluon correlation: $T_F(x,x) \propto \overline{\psi}_i F^C (T^C)_{ij} \psi_j$

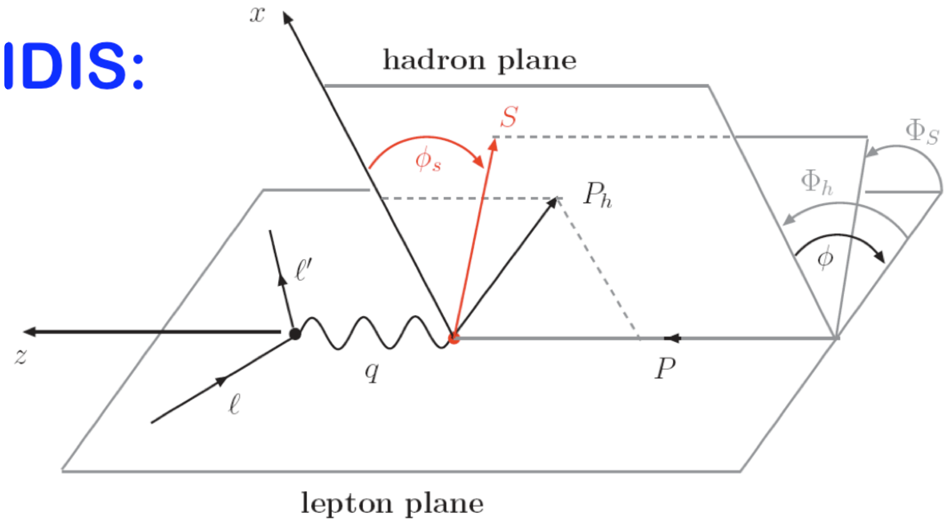
- D-meson production in SIDIS:

  - Clean probe for gluonic twist-3 correlation functions
  - $T_G^{(f)}(x,x)$ could be connected to the gluonic Sivers function

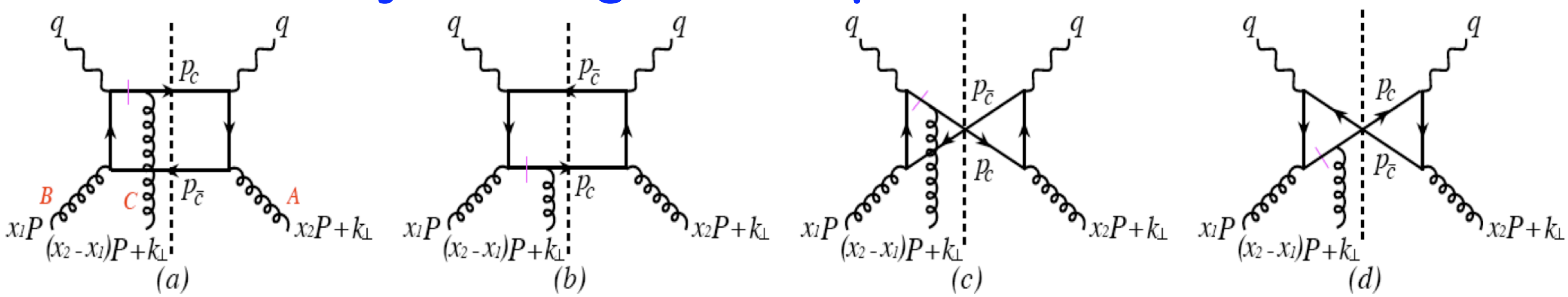
D-meson production in SIDIS

Kang, Qiu, PRD 2008

❑ Frame for SIDIS:



❑ Dominated by the tri-gluon subprocess:



❑ Single transverse-spin asymmetry:

$$A_N = \frac{\sigma(s_\perp) - \sigma(-s_\perp)}{\sigma(s_\perp) + \sigma(-s_\perp)} = \frac{d\Delta\sigma(s_\perp)}{dx_B dy dz_h dP_{h\perp}^2 d\phi} \Bigg/ \frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 d\phi}$$

Spin-dependent cross section

- Contribution from $T_G^{(f)}$:

Color factor

$$\frac{d\Delta\sigma(s_\perp)}{dx_B dy dz_h dP_{h\perp}^2 d\phi} = \sigma_0 \int_{x_{min}}^1 dx \int \frac{dz}{z} D(z) \delta\left(\frac{P_{h\perp}^2}{z_h^2} - \frac{(1-\hat{x})(1-\hat{z})}{\hat{x}\hat{z}} Q^2 + \hat{z}^2 m_c^2\right)\left(\frac{1}{4}\right)$$
$$\times \left[\epsilon^{P_h s_\perp n \bar{n}} \left(\frac{\sqrt{4\pi\alpha_s}}{z\hat{t}}\right)\left(1+\frac{\hat{t}}{\hat{u}}\right)\right] \sum_{i=1}^4 \mathcal{A}_i \left[-x\frac{d}{dx}\left(\frac{T_G(x,x)}{x}\right)\hat{W}_i + \left(\frac{T_G(x,x)}{x}\right)\hat{N}_i\right]$$

- Partonic hard parts: $\hat{W}_i \neq \hat{N}_i$

$$\mathcal{A}_1 = 1+\cosh^2\psi \quad \mathcal{A}_2 = -2 \quad \mathcal{A}_3 = -\cos\phi\sinh 2\psi \quad \mathcal{A}_4 = \cos 2\phi \sinh^2\psi$$

$$\cosh\psi = \frac{2x_B S_{ep}}{Q^2} - 1 = \frac{2}{y} - 1$$

$$\hat{W}_1 = 2\left[\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} - \frac{2\hat{s}Q^2}{\hat{t}\hat{u}} + \frac{4\hat{x}^2\hat{s}}{Q^2}\right] + 4m_c^2\left[\frac{Q^2-2\hat{t}}{\hat{t}^2} + \frac{Q^2-2\hat{u}}{\hat{u}^2} - \frac{2\hat{x}^2}{Q^2}\left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + 2\right)\right] - 8m_c^4\left[\frac{1}{\hat{t}} + \frac{1}{\hat{u}}\right]^2$$

$$\hat{W}_2 = \frac{16\hat{x}^2}{Q^2}\left[\hat{s} - m_c^2\left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + 2\right)\right],$$

$$\hat{W}_3 = 4\hat{x}\hat{z}\frac{q_\perp}{Q}(\hat{u}-\hat{t})\left[\frac{\hat{s}-Q^2}{\hat{t}\hat{u}} - 2m_c^2\left(\frac{1}{\hat{t}} + \frac{1}{\hat{u}}\right)^2\right]$$

$$\hat{W}_4 = 8\hat{z}^2 q_\perp^2\left[\frac{Q^2}{\hat{t}\hat{u}} + m_c^2\left(\frac{1}{\hat{t}} + \frac{1}{\hat{u}}\right)^2\right],$$

$$\hat{N}_1 = 4\left[\frac{2m_c^2 - Q^2}{\hat{t}\hat{u}} + \frac{6\hat{x}^2}{Q^2}\right]\left[(\hat{s}-Q^2) - 2m_c^2\left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + 2\right)\right],$$

$$\hat{N}_2 = \frac{16\hat{x}^2}{Q^2}\left[(\hat{s}-Q^2) - 2m_c^2\left(\frac{\hat{u}}{\hat{t}} + \frac{\hat{t}}{\hat{u}} + 2\right)\right],$$

$$\hat{N}_3 = \frac{2Q}{\hat{z}q_\perp}(\hat{u}-\hat{t})\left[\left(\frac{4\hat{z}^2 q_\perp^2}{\hat{t}\hat{u}} - \frac{1}{Q^2+\hat{s}}\right)\left(2m_c^2\left(\frac{1}{\hat{t}} + \frac{1}{\hat{u}}\right) - \frac{Q^2-\hat{s}}{Q^2+\hat{s}}\right) - 2\hat{z}q_\perp^2\right]$$

$$\hat{N}_4 = 8\left[2\hat{z}q_\perp^2 - \frac{\hat{t}\hat{u}}{Q^2+\hat{s}}\right]\left[\frac{Q^2}{\hat{t}\hat{u}} + m_c^2\left(\frac{1}{\hat{t}} + \frac{1}{\hat{u}}\right)^2\right].$$

- Contribution from $T_G^{(d)}$ is the same except the color factor

Features of the SSA

- Dependence on tri-gluon correlation functions:

$$D-\text{meson} \propto T_G^{(f)} + T_G^{(d)} \qquad \overline{D}-\text{meson} \propto T_G^{(f)} - T_G^{(d)}$$

Separate $T_G^{(f)}$ and $T_G^{(d)}$ by the difference between D and $\bar{D}$

- Model for tri-gluon correlation functions:

$$T_G^{(f,d)}(x,x) = \lambda_{f,d} G(x) \qquad \lambda_{f,d} = \pm\lambda_F = \pm 0.07\text{GeV}$$

- Kinematic constraints:

$$x_{min} = \begin{cases} x_B\left[1+\frac{P_{h\perp}^2+m_c^2}{z_h(1-z_h)Q^2}\right], & \text{if } z_h+\sqrt{z_h^2+\frac{P_{h\perp}^2}{m_c^2}} \geq 1 \\ x_B\left[1+\frac{2m_c^2}{Q^2}\left(1+\sqrt{1+\frac{P_{h\perp}^2}{z_h^2 m_c^2}}\right)\right], & \text{if } z_h+\sqrt{z_h^2+\frac{P_{h\perp}^2}{m_c^2}} \leq 1 \end{cases}$$

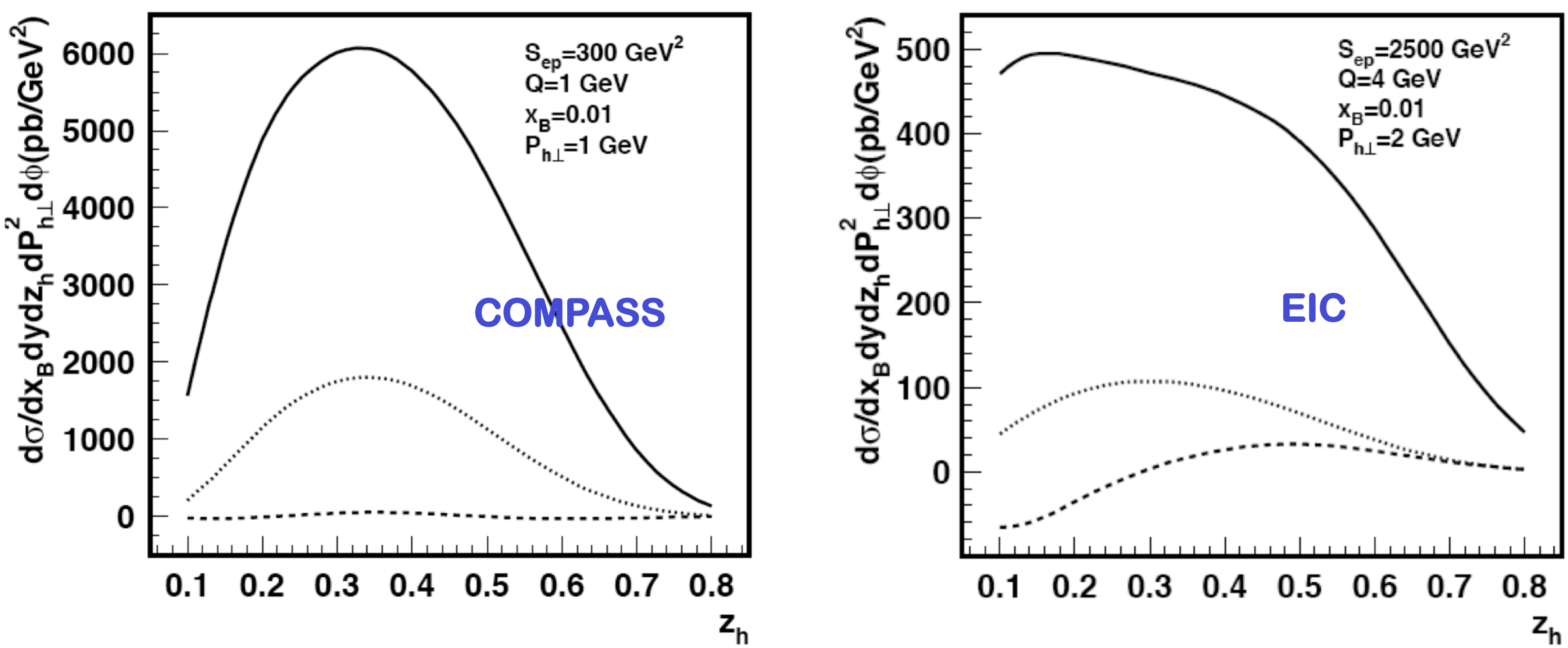
Note: The $z_h(1-z_h)$ has a maximum

SSA should have a minimum if the derivative term dominates

Numerical Estimate

❑ Production rate (spin averaged):

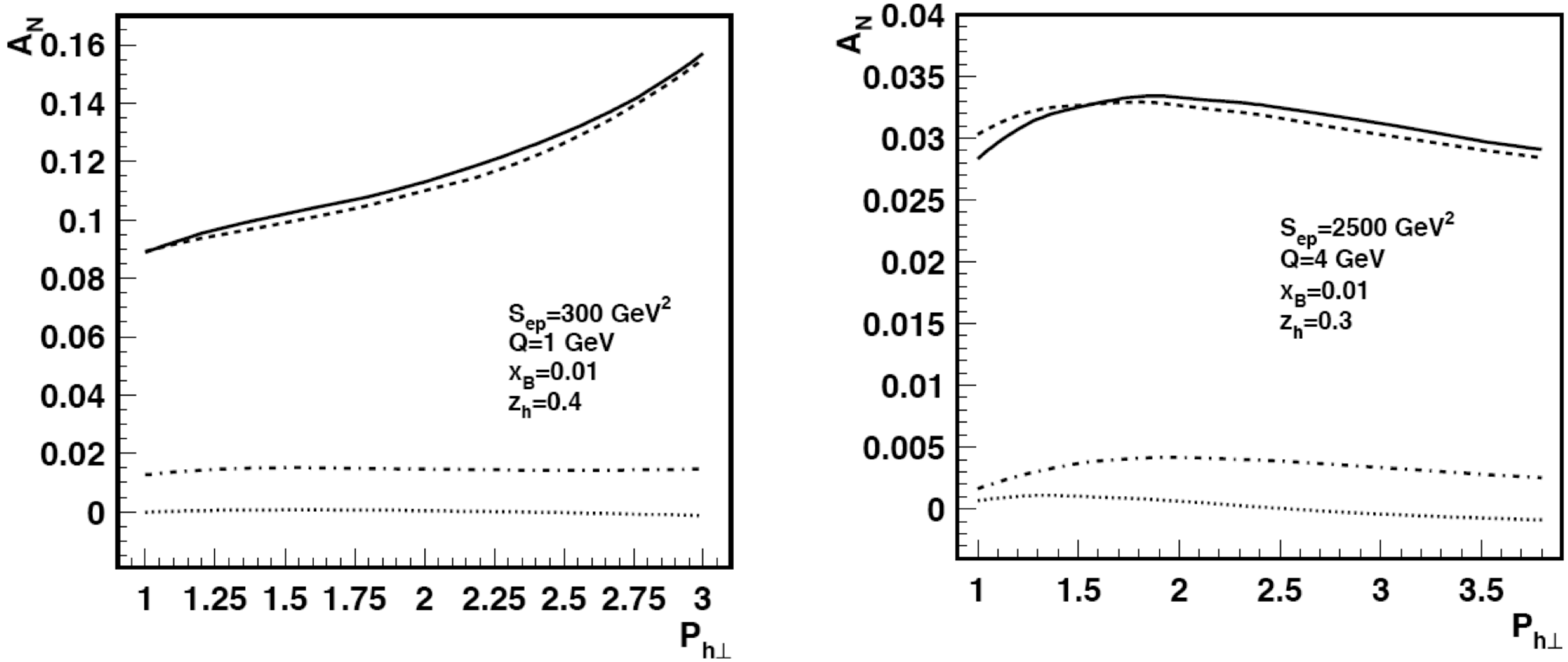
$$\frac{d\sigma}{dx_B dy dz_h dP_{h\perp}^2 \, d\phi} = \sigma_0^U + \sigma_1^U \cos\phi + \sigma_2^U \cos 2\phi$$



Small φ dependence, reasonable production rate

Maximum in the SSA of D-production

- SSA for D^0 production ($T_G^{(f)}$ only):



- The SSA is a twist-3 effect, it should fall off as $1/P_T$ when $P_T >> m_c$
- For the region, $P_T \sim m_c$,

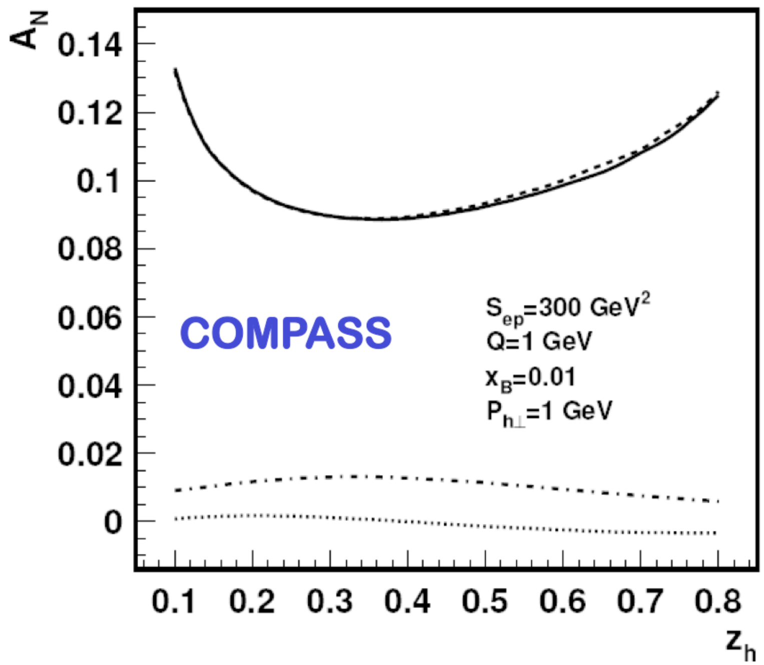
$$A_N \propto \epsilon^{P_h s_\perp n \bar{n}} \frac{1}{\tilde{t}} = -\sin\phi_s \frac{P_{h\perp}}{\tilde{t}}$$

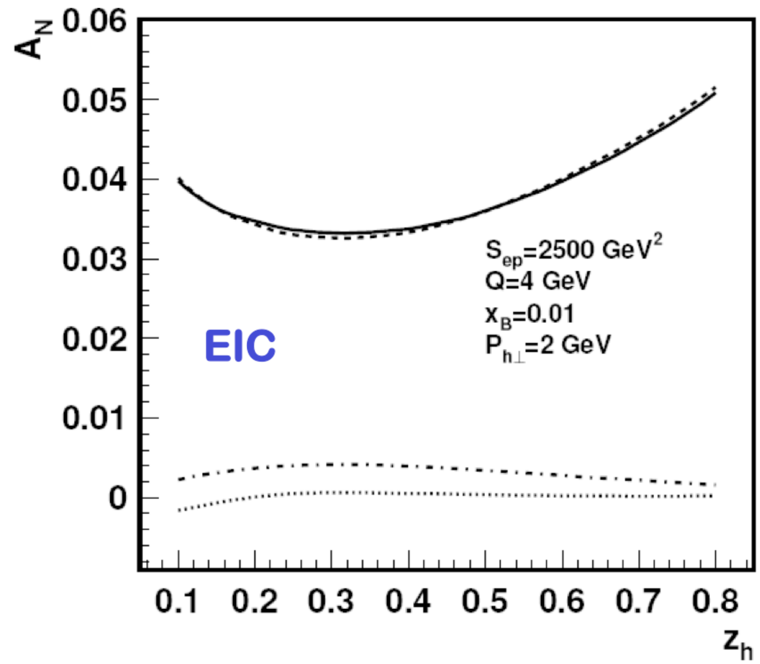
$$\tilde{t} = (p_c - q)^2 - m_c^2 = -\frac{1-\hat{z}}{\hat{x}} Q^2$$

$$\hat{z} = z_h/z, \quad \hat{x} = x_B/x$$

Minimum in the SSA of D-production

- SSA for D^0 production ($T_G^{(f)}$ only):

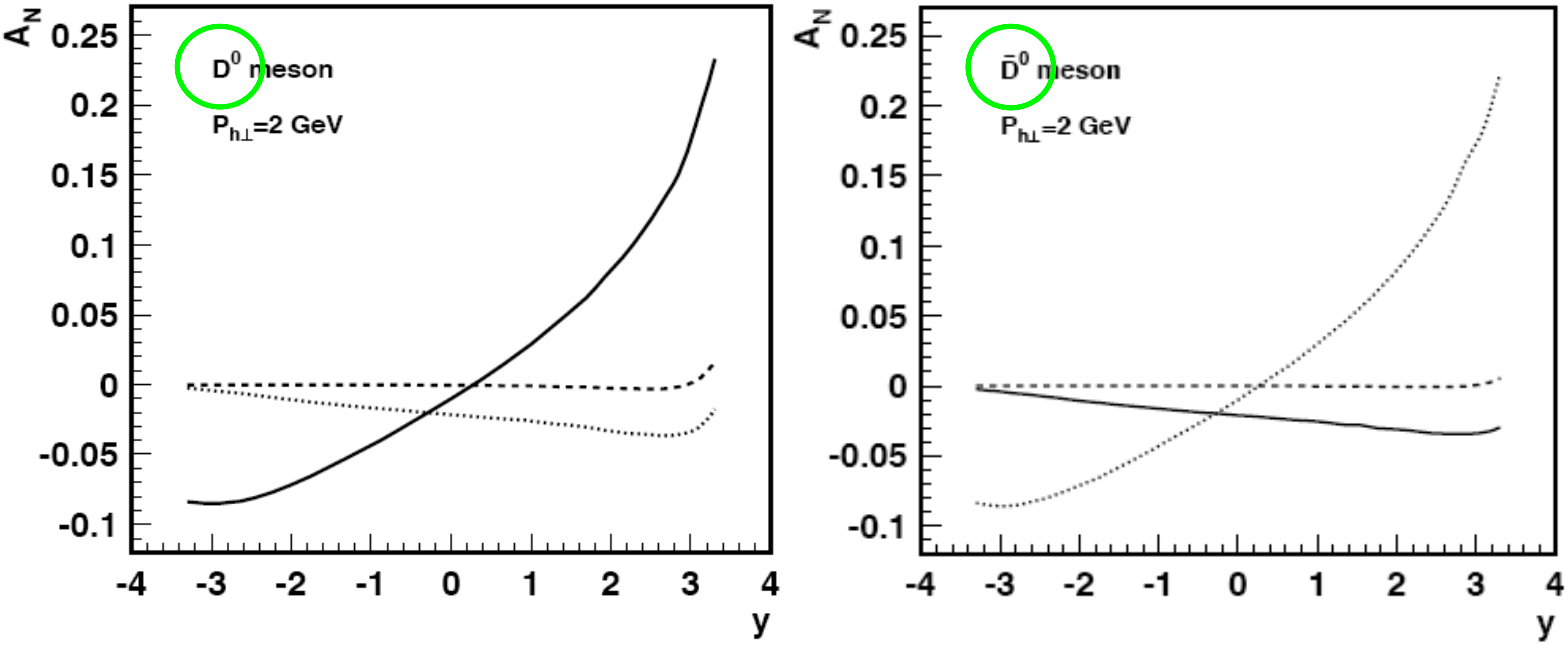




- Derivative term dominates, and small φ dependence
- Asymmetry is **twice** if $T_G^{(f)} = +T_G^{(d)}$, or **zero** if $T_G^{(f)} = -T_G^{(d)}$
- If $T_G^{(d)} = 0$, same SSA for $\bar{D}$ meson.
- Asymmetry has a minimum ~ z_h ~ 0.5

SSA of D-production in Hadronic Collisions

- **SSA at RHIC:** $\sqrt{s} = 200\ \text{GeV}$ $\quad \mu = \sqrt{m_c^2 + P_{h\perp}^2}$ $\quad m_c = 1.3\ \text{GeV}$



Dashed: $(2)\ \lambda_f = \lambda_d = 0$ $\qquad T_G^{(f)} = T_G^{(d)} = 0$

Solid: $(1)\ \lambda_f = \lambda_d = 0.07\ \text{GeV}$ $\qquad T_G^{(f)} = T_G^{(d)}$

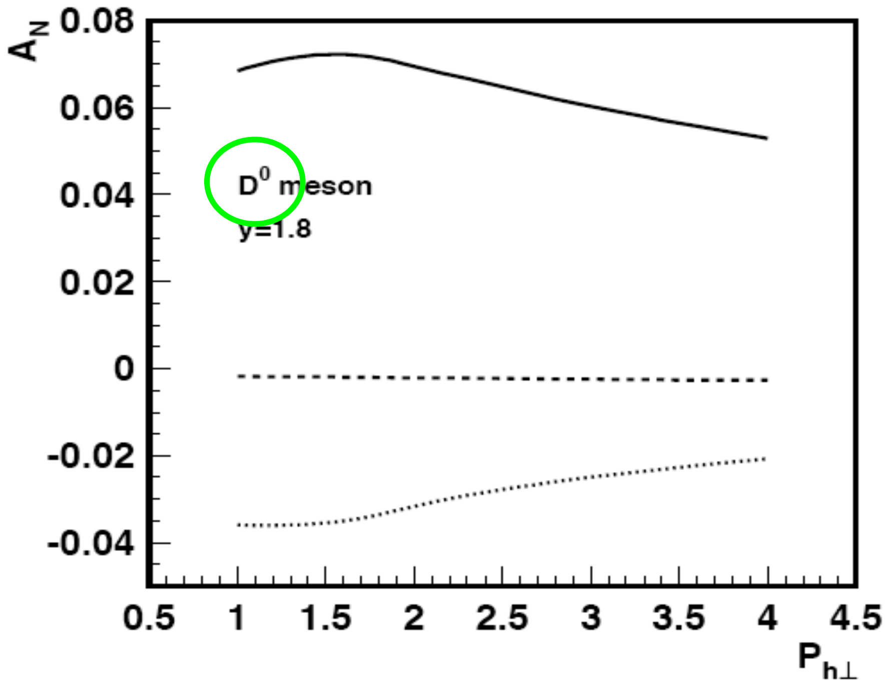
Dotted: $(3)\ \lambda_f = -\lambda_d = 0.07\ \text{GeV}$ $\qquad T_G^{(f)} = -T_G^{(d)}$

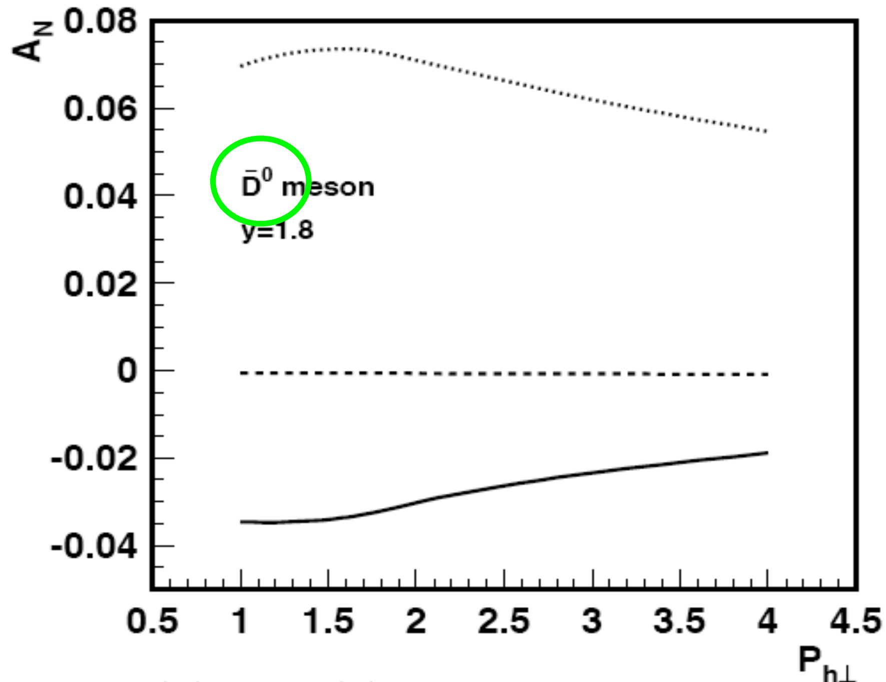
Any sizable SSA = tri-gluon correlation

Kang, Qiu, Vogelsang, Yuan, 2008

Maximum of the SSA

- **SSA at RHIC:** $\sqrt{s} = 200\ \text{GeV}$ $\mu = \sqrt{m_c^2 + P_{h\perp}^2}$ $m_c = 1.3\ \text{GeV}$





Dashed: (2) $\lambda_f = \lambda_d = 0$ $\quad T_G^{(f)} = T_G^{(d)} = 0$

Solid: (1) $\lambda_f = \lambda_d = 0.07\ \text{GeV}$ $\quad T_G^{(f)} = T_G^{(d)}$

Dotted: (3) $\lambda_f = -\lambda_d = 0.07\ \text{GeV}$ $\quad T_G^{(f)} = -T_G^{(d)}$

SSA decreases when $p_T \gg m_c$

Evolution Equations for Tri-gluon Correlation

Kang, Qiu, 0811.3101 [hep-ph]

$$
\begin{aligned}
\frac{\partial T^{(f)}_{G,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Big\{ & P_{gg}(z)\, T_G^{(f)}(\xi,\xi,\mu_F) \\
& + \frac{C_A}{2}\left[2\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)\left[T^{(f)}_{G,F}(\xi,x,\mu_F)-T^{(f)}_{G,F}(\xi,\xi,\mu_F)\right]\right. \\
& \qquad \left. +2\left(1-\frac{1-z}{2z}-z(1-z)\right)T^{(f)}_{G,F}(\xi,x,\mu_F)\right] \\
& + \frac{C_A}{2}\left[(1+z)\, T^{(f)}_{\Delta G,F}(x,\xi,\mu_F)\right] \\
& + P_{gq}(z)\left(\frac{N_c^2}{N_c^2-1}\right)\sum_q \left[T_{q,F}(\xi,\xi,\mu_F)-T_{\bar{q},F}(\xi,\xi,\mu_F)\right]\Big\};
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial T^{(d)}_{G,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Big\{ & P_{gg}(z)\, T^{(d)}_{G,F}(\xi,\xi,\mu_F) \\
& + \frac{C_A}{2}\left[2\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)\left[T^{(d)}_{G,F}(\xi,x,\mu_F)-T^{(d)}_{G,F}(\xi,\xi,\mu_F)\right]\right. \\
& \left. +2\left(1-\frac{1-z}{2z}-z(1-z)\right)T^{(d)}_{G,F}(\xi,x,\mu_F)\right] \\
& + \frac{C_A}{2}\left[(1+z)\, T^{(d)}_{\Delta G,F}(x,\xi,\mu_F)\right] \\
& + P_{gq}(z)\left(\frac{N_c^2-4}{N_c^2-1}\right)\sum_q \left[T_{q,F}(\xi,\xi,\mu_F)+T_{\bar{q},F}(\xi,\xi,\mu_F)\right]\Big\}.
\end{aligned}
$$

- ✧ **Similar to DGLAP of PDFs, All kernels are IR safe**
- ✧ $T_G^{(d)}$ **can be perturbatively generated if** $T_{q,F}+T_{\bar{q},F}\neq 0$

Scale dependence of Quark-gluon Correlation

Kang, Qiu, 2008

$$
\begin{aligned}
\frac{\partial T_{q,F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Big\{ & P_{qq}(z)\,T_{q,F}(\xi,\xi,\mu_F) \\
& + \frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{q,F}(\xi,x,\mu_F) - T_{q,F}(\xi,\xi,\mu_F)\right] + z\,T_{q,F}(\xi,x,\mu_F)\right] \\
& + \frac{C_A}{2}\left[T_{\Delta q,F}(x,\xi,\mu_F)\right] \\
& + P_{qg}(z)\left(\frac{1}{2}\right)\left[T^{(d)}_{G,F}(\xi,\xi,\mu_F) + T^{(f)}_{G,F}(\xi,\xi,\mu_F)\right]\Big\} \\
\frac{\partial T_{\bar{q},F}(x,x,\mu_F)}{\partial \ln \mu_F^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{d\xi}{\xi}\Big\{ & P_{qq}(z)\,T_{\bar{q},F}(\xi,\xi,\mu_F) \\
& + \frac{C_A}{2}\left[\frac{1+z^2}{1-z}\left[T_{\bar{q},F}(\xi,x,\mu_F) - T_{\bar{q},F}(\xi,\xi,\mu_F)\right] + z\,T_{\bar{q},F}(\xi,x,\mu_F)\right] \\
& + \frac{C_A}{2}\left[T_{\Delta \bar{q},F}(x,\xi,\mu_F)\right] \\
& + P_{qg}(z)\left(\frac{1}{2}\right)\left[T^{(d)}_{G,F}(\xi,\xi,\mu_F) - T^{(f)}_{G,F}(\xi,\xi,\mu_F)\right]\Big\};
\end{aligned}
$$

✧ **Equations depend on off-diagonal correlation functions**

✧ **also on a new set of twist-3 correlation functions**

Twist-3 correlation functions relevant to SSAs

❑ Set I:

Qiu, Sterman, 1991, 1998
Ji, 1992, Kang, Qiu, 2008

$$\widetilde{T}_{q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P, s_T | \overline{\psi}_q(0) \frac{\gamma^+}{2} \left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right] \psi_q(y_1^-) | P, s_T \rangle \qquad q(x)$$

$$\widetilde{T}^{(f,d)}_{G,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P^+} \langle P, s_T | F^{+\rho}(0) \left[\epsilon^{s_T\sigma n\bar{n}} F_\sigma{}^+(y_2^-)\right] F^{+\lambda}(y_1^-) | P, s_T \rangle (-g_{\rho\lambda}) \qquad xG(x)$$

❑ Set II:

$$\widetilde{T}_{\Delta q,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \langle P, s_T | \overline{\psi}_q(0) \frac{\gamma^+\gamma^5}{2} \left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right] \psi_q(y_1^-) | P, s_T \rangle \qquad \Delta q(x)$$

$$\widetilde{T}^{(f,d)}_{\Delta G,F} = \int \frac{dy_1^- dy_2^-}{(2\pi)^2} e^{ixP^+y_1^-} e^{ix_2P^+y_2^-} \frac{1}{P^+} \langle P, s_T | F^{+\rho}(0) \left[i\, s_T^\sigma\, F_\sigma{}^+(y_2^-)\right] F^{+\lambda}(y_1^-) | P, s_T \rangle (i\epsilon_{\perp\rho\lambda}) \qquad x\Delta G(x)$$

Two possible color contractions: if_{abc}, d_{abc}
Two possible tri-gluon correlation functions

Connection to Twist-2 PDFs

- **Set I:**

  Spin-averaged twist-2 PDFs + an operator Insertion

$$\int \frac{dy_2^-}{2\pi}\, e^{ix_2 P^+ y_2^-}\, \left[\epsilon^{s_T \sigma n \bar{n}}\, F_\sigma{}^+(y_2^-)\right] = i \int \frac{dy_2^-}{2\pi}\, e^{ix_2 P^+ y_2^-}\, \left[i\, \epsilon_\perp^{\rho\sigma}\, s_{T\rho}\, F_\sigma{}^+(y_2^-)\right]$$

- **Set II:**

  Spin-dependent twist-2 HDFs + an operator Insertion

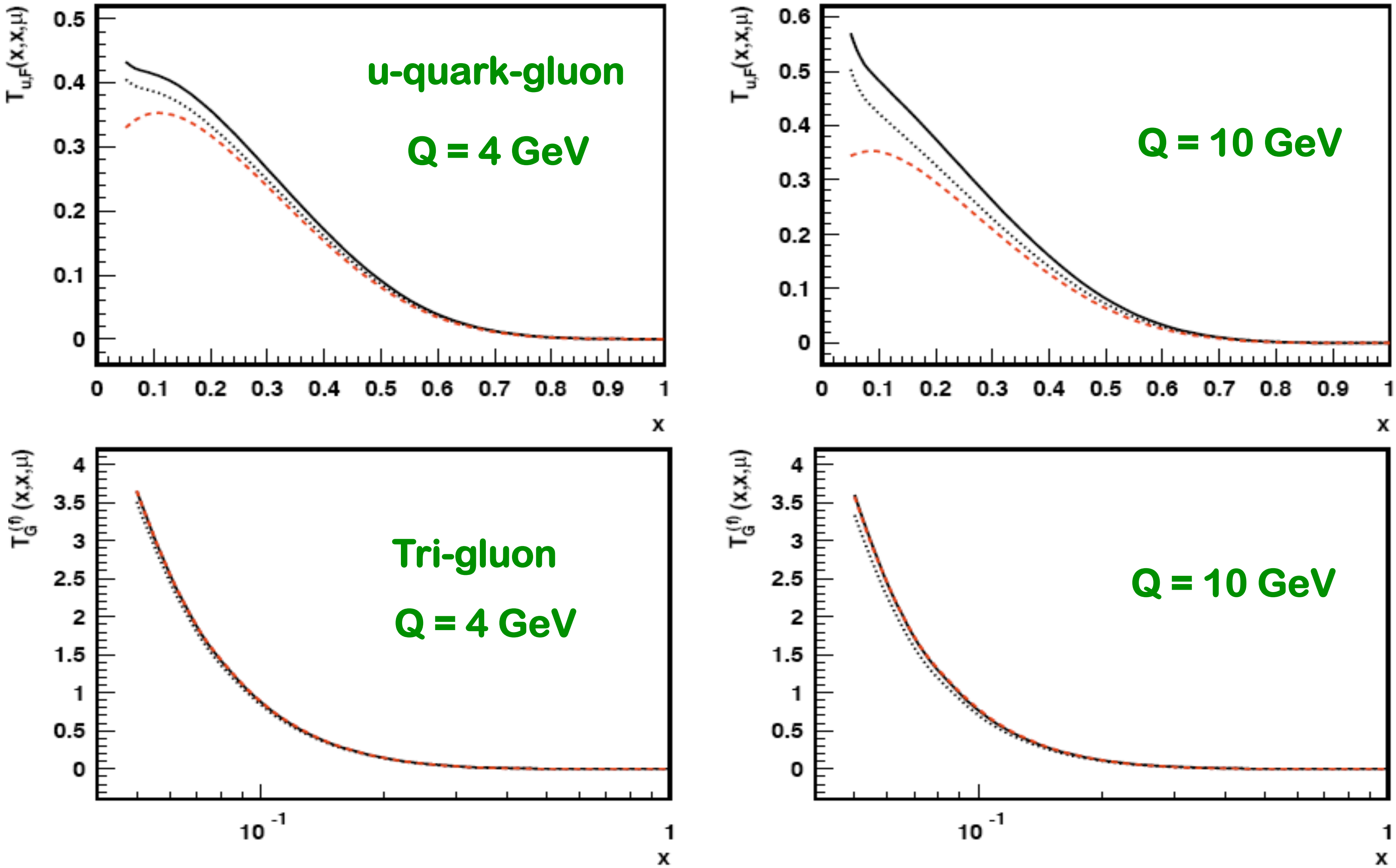
$$i \int \frac{dy_2^-}{2\pi}\, e^{ix_2 P^+ y_2^-}\, \left[s_T^\sigma\, F_\sigma{}^+(y_2^-)\right]$$

- **Extra ‘i’**

  Phase needed for the nonvanishing SSAs

  Do not contribute to parity conserving double-spin asymmetry! such as g_2

Q^2-Dependence of Correlation Functions



Similar to DGLAP except small-x region

Summary

- Single transverse-spin asymmetry is directly connected to the parton's transverse motion (P and T invariance)

  – an excellent probe for the cause of parton's transverse motion

- Two complementary approaches:

  TMD: direct k_T information – two-scale observables

  Collinear: net spin-dependence of all k_T – single-scale observalbes

- D-meson production in SIDIS, as well as in hadron-hadron collisions, is an excellent observable to measure the new tri-gluon correlation functions

  → QCD global analysis of twist-3 distributions that are responsible for the SSAs $T_{q,F}$ $T_{G,F}$ $T_{\Delta q,F}$ $T_{\Delta G,F}$

Backup slides

What is the $T_F(x,x)$?

- Twist-3 correlation $T_F(x,x)$:

$$T_F(x,x) = \int \frac{dy_1^-}{4\pi} e^{ixP^+y_1^-} \times \langle P, \vec{s}_T | \bar{\psi}_a(0)\gamma^+ \left[\int dy_2^- \, \epsilon^{s_T \sigma n \bar{n}} \, F_\sigma^{\ +}(y_2^-) \right] \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

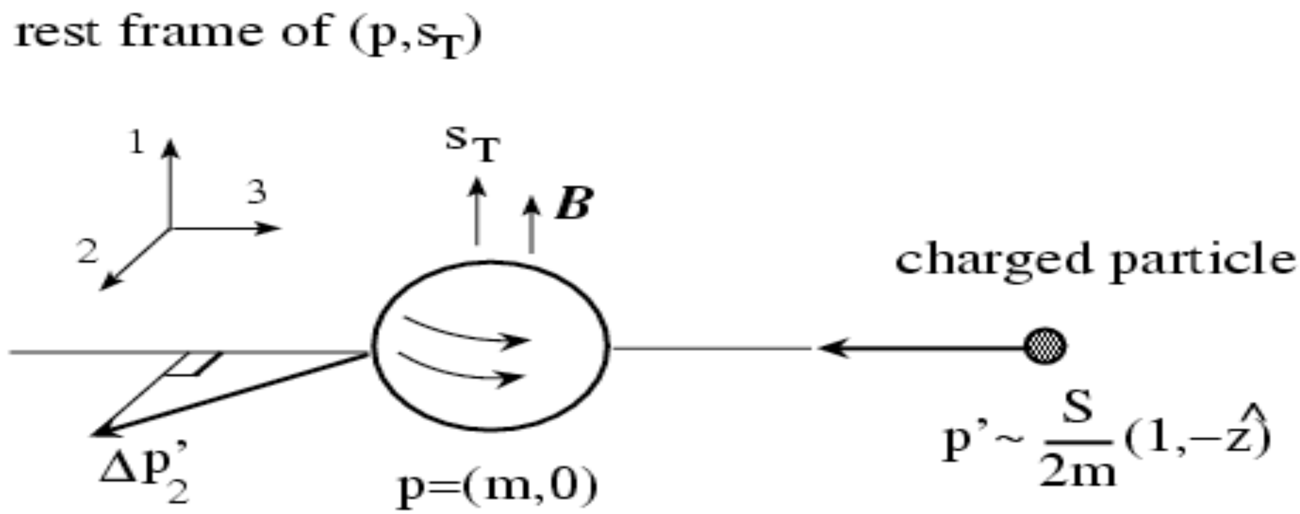
- Twist-2 quark distribution:

$$q(x) = \int \frac{dy_1^-}{4\pi} e^{ixP^+y_1^-} \langle P, \vec{s}_T | \bar{\psi}_a(0)\gamma^+ \psi_a(y_1^-) | P, \vec{s}_T \rangle$$

$T_F(x,x)$ represents a net spin dependence of a quark's transverse motion via a gluon interaction inside a transversely polarized proton

What the $T_F(x,x)$ tries to tells us?

- Consider a classical (Abelian) situation:



– change of transverse momentum

$$\frac{d}{dt}p_2' = e(\vec{v}' \times \vec{B})_2 = -ev_3 B_1 = ev_3 F_{23}$$

– in the c.m. frame

$$(m, \vec{0}) \to \bar{n} = (1, 0, 0_T), \quad (1, -\hat{z}) \to n = (0, 1, 0_T)$$

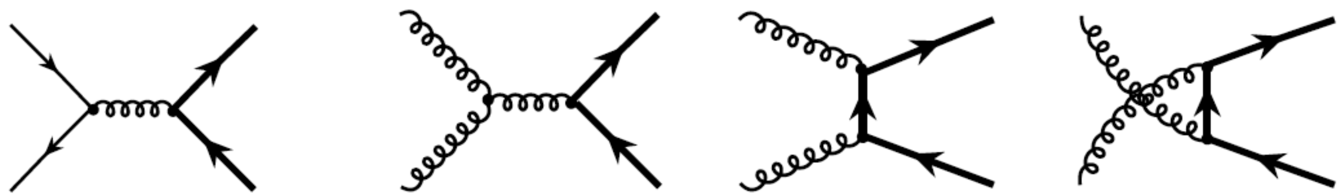
$$\Longrightarrow \frac{d}{dt}p_2' = e\,\epsilon^{s_T \sigma n \bar{n}}\, F_\sigma^{\ +}$$

– total change: $\boxed{\Delta p_2' = e \int dy^- \epsilon^{s_T \sigma n \bar{n}}\, F_\sigma^{\ +}(y^-)}$

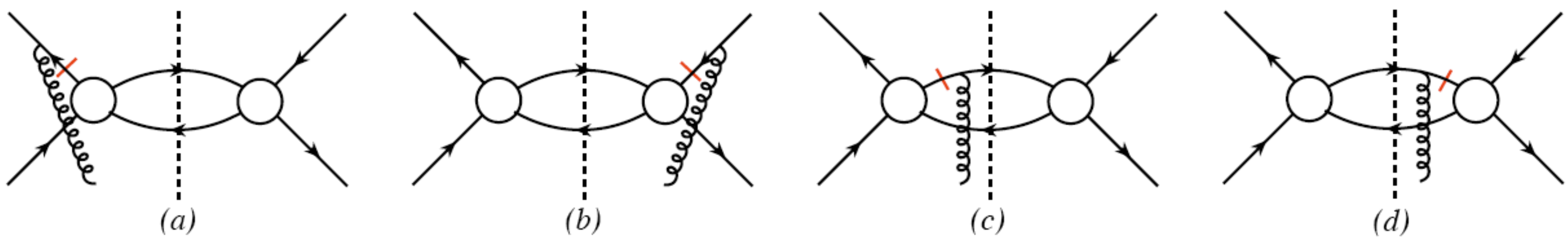
D-meson production in Hadronic Collisions

Kang, Qiu, Vogelsang, Yuan, 2008

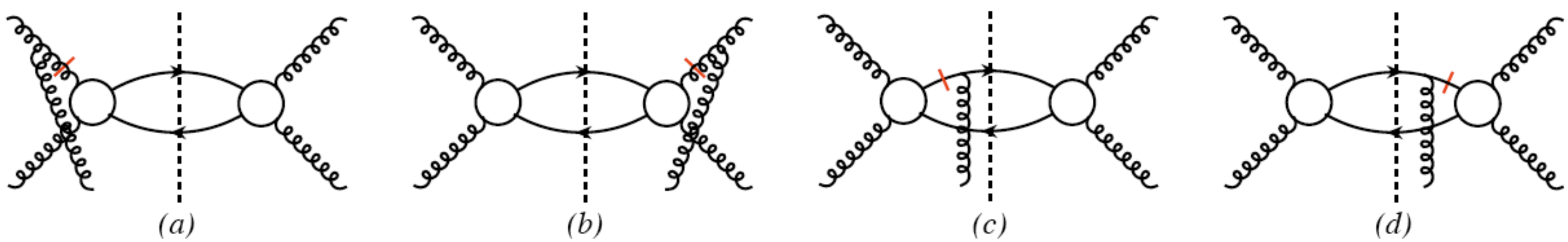
- Two partonic subprocesses:



- Quark-antiquark annihilation:



- Gluon-gluon fusion:



Factorized formula for D-meson production

- Same factorized formula for both subprocesses:

$$E_{P_h}\frac{d\Delta\sigma}{d^3P_h}\bigg|_{q\bar{q}\to c\bar{c}} = \frac{\alpha_s^2}{S}\sum_q\int\frac{dz}{z^2}D_{c\to h}(z)\int\frac{dx'}{x'}\phi_{\bar{q}/B}(x')\int\frac{dx}{x}\sqrt{4\pi\alpha_s}\left(\frac{\epsilon^{P_h s_T n\bar{n}}}{z\tilde{u}}\right)\delta\left(\tilde{s}+\tilde{t}+\tilde{u}\right)$$
$$\times\left[\boxed{\left(T_{q,F}(x,x)-x\frac{d}{dx}T_{q,F}(x,x)\right)}H_{q\bar{q}\to c}(\tilde{s},\tilde{t},\tilde{u})+T_{q,F}(x,x)\mathcal{H}_{q\bar{q}\to c}(\tilde{s},\tilde{t},\tilde{u})\right],$$

$$E_{P_h}\frac{d\Delta\sigma}{d^3P_h}\bigg|_{gg\to c\bar{c}} = \frac{\alpha_s^2}{S}\sum_{i=f,d}\int\frac{dz}{z^2}D_{c\to h}(z)\int\frac{dx'}{x'}\phi_{g/B}(x')\int\frac{dx}{x}\sqrt{4\pi\alpha_s}\left(\frac{\epsilon^{P_h s_T n\bar{n}}}{z\tilde{u}}\right)\delta\left(\tilde{s}+\tilde{t}+\tilde{u}\right)$$
$$\times\left[\boxed{\left(T_G^{(i)}(x,x)-x\frac{d}{dx}T_G^{(i)}(x,x)\right)}H^{(i)}_{gg\to c}(\tilde{s},\tilde{t},\tilde{u})+T_G^{(i)}(x,x)\mathcal{H}^{(i)}_{gg\to c}(\tilde{s},\tilde{t},\tilde{u})\right],$$

- Hard parts:

$$H_{q\bar{q}\to c}=H^I_{q\bar{q}\to c}+H^F_{q\bar{q}\to c}\left(1+\frac{\tilde{u}}{\tilde{t}}\right)\qquad H^{(i)}_{gg\to c}=H^{I(i)}_{gg\to c}+H^{F(i)}_{gg\to c}\left(1+\frac{\tilde{u}}{\tilde{t}}\right)$$

All $\mathcal{H}_{q\bar{q}\to c}$ and $\mathcal{H}^{I(i)}_{gg\to c}$ and $\mathcal{H}^{F(i)}_{gg\to c}$ vanish as $m_c^2\to 0$

- Hard parts change sign for $T_G^{(d)}(x,x)$ when $c\longrightarrow\bar{c}$

$$H^{(f)}_{gg\to\bar{c}}=H^{(f)}_{gg\to c},\qquad H^{(d)}_{gg\to\bar{c}}=-H^{(d)}_{gg\to c},$$
$$\mathcal{H}^{(f)}_{gg\to\bar{c}}=\mathcal{H}^{(f)}_{gg\to c},\qquad \mathcal{H}^{(d)}_{gg\to\bar{c}}=-\mathcal{H}^{(d)}_{gg\to c}.$$