Semi-inclusive production: low p_T, high p_T, and in between

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LBNL, 11 December 2008

present results from

A. Bacchetta, D. Boer, M. Diehl, P. Mulders, JHEP 0808 (2008) 023

closely related to work: X. Ji, J.-W. Qiu, W. Vogelsang, F. Yuan '06 and Y. Koike, W. Vogelsang, F. Yuan '07 on Sivers asymmetry

1. The question
2. High-q_T results
3. Low-q_T results
4. Comparison
5. Integrated observables
6. Summary

warning: will use power counting as main tool

- – no numerical estimates at this stage
- + aim at clarifying theoretical situation and inducing further work
- + consequences for experimental analyses

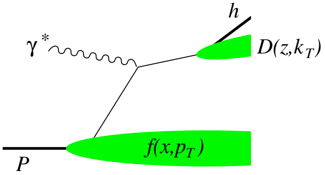
The physics question:

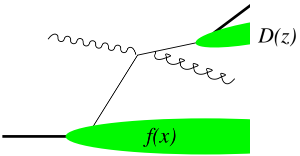
- general setting: hard processes with measured transverse momentum $\boldsymbol{q}_T$ in the final state
- here: semi-inclusive deep inelastic scattering

  $$ep \to e + h + X$$

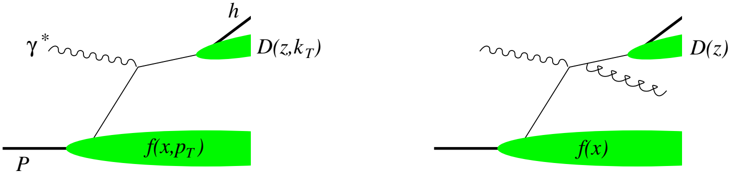
  transfer results to
  - Drell-Yan process $pp \to \ell^+\ell^- + X$
  - hadron pair production $e^+e^- \to h_1 + h_1 + X$

  by crossing symmetry
- physics motivations:
  - understand a basic feature of QCD final states
  - use as tool for extracting specific parton distributions
- two different frameworks to describe $\boldsymbol{q}_T$ distribution $\rightsquigarrow$





- 'intrinsic transverse momentum' of partons in hadron
  use p_T dependent parton densities and fragmentation fcts.
  adequate for low q_T Cahn '78, 89
- perturbative radiation
  use standard collinear pdfs and fragm. fcts.
  adequate for high q_T Georgi, Politzer '78
- both mechanisms $\rightsquigarrow$ nontrivial angular dependence of $\boldsymbol{q}_T$
  nonzero cross section for longitudinal γ^*



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- **How are the two mechanisms related?**
  complementary? try to interpolate between them?
  competing? can add without double counting?

Scales and power counting

photon virtuality Q, transv. mom. q_T, nonperturbative scale M
in following always $Q \gg M$

- "low-q_T" mechanism for $q_T \ll Q$
  twist expansion in powers of q_T/Q
- "high-q_T" mechanism for $q_T \gg M$
  twist expansion in powers of M/q_T
- in intermediate region $M \ll q_T \ll Q$ can use both descript's and further expand in second ratio of scales
- consistency $\rightsquigarrow$ full results must coincide
- in practice can only calculate leading nonzero terms in each approach (twist 2 and possibly twist 3) and these need not coincide

Scales and power counting: two examples

- $$\mathcal{O}(q_T, Q) = A\, \frac{M^2}{M^2 + q_T^2} + B\, \frac{q_T^2}{Q^2}\, \frac{M^2}{M^2 + q_T^2}$$
$$\overset{q_T \ll Q}{=} A\, \frac{M^2}{M^2 + q_T^2} + \mathcal{O}\Big(\frac{q_T^2}{Q^2}\Big)$$
$$\overset{M \ll q_T}{=} A\, \frac{M^2}{q_T^2} + B\, \frac{M^2}{Q^2} + \mathcal{O}\Big(\frac{M^4}{q_T^4}\Big)$$

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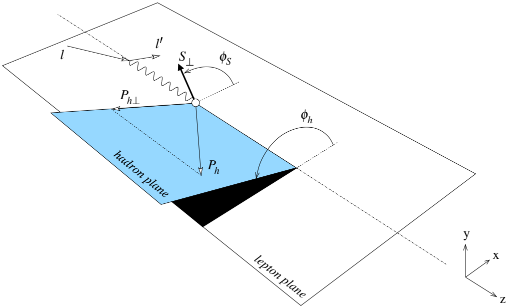
$\rightsquigarrow$ leading-order terms in both calculations differ for $M \ll q_T \ll Q$
consistency only explicit if retain higher-order terms in each approx.

Variables and observables for $ep \to e + h + X$

- photon virtuality Q^2, photon polarization parameter ε
- Bjorken x and z
- hadron transverse momentum $P_{h\perp}$ ... use $q_T = P_{h\perp}/z$
- azimuth ϕ between hadron and lepton plane
  and azimuth ϕ_S for transverse target polarization



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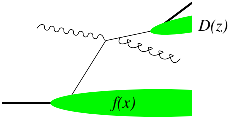
decompose ep cross section as

$$\frac{d\sigma(ep \to ehX)}{d\phi\, d \ldots} = \text{(kin. factor)}$$
$$\times \Big[F_{UU,T} + \varepsilon F_{UU,L} + 2\sqrt{\varepsilon(1+\varepsilon)} \cos\phi\, F_{UU}^{\cos\phi} + \varepsilon \cos 2\phi\, F_{UU}^{\cos 2\phi} \Big]$$
$$+ S_T \Big[\sin(\phi - \phi_S)\, F_{UT,T}^{\sin(\phi-\phi_S)} + \cdots \Big] + \cdots$$

$\rightsquigarrow$ semi-inclusive structure functions $F_{\cdots}^{\cdots}(x, z, Q, q_T)$

Results of the high-q_T calculation

$$F^{\cdots}_{\cdots} = \frac{1}{q_T^2} \sum_{\text{partons } i,j} \int \frac{d\hat{x}}{\hat{x}} \int \frac{d\hat{z}}{\hat{z}}\, f_1^i\Big(\frac{x}{\hat{x}}\Big)\, D_1^j\Big(\frac{z}{\hat{z}}\Big)\, K^{ij}_{\cdots}\Big(\hat{x}, \hat{z}, \frac{q_T}{Q}\Big)$$



expanding hard-scattering kernels $K^{ij}_{\cdots}$ for $\frac{q_T}{Q} \to 0$ find

$$F_{UU,T} \sim \frac{1}{q_T^2} \qquad F_{UU,L} \sim \frac{1}{Q^2}$$

$$F_{UU}^{\cos\phi} \sim \frac{1}{Q\, q_T} \qquad F_{UU}^{\cos 2\phi} = \frac{1}{2} F_{UU,L}$$

- all struct. fcts. have a term $\propto f_1(x)\, D_1(z)\, \log \frac{Q^2}{q_T^2}$
  higher orders give $\alpha_s^n \log^{2n-1}(Q/q_T)$ $\rightsquigarrow$ should resum

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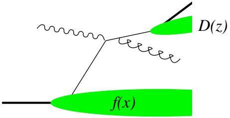

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- analogous for F_{LL} and $F_{LL}^{\cos\phi}$ with $f_1^i \to g_1^i$

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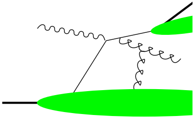

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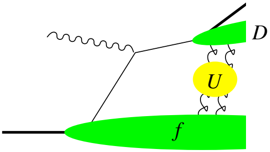
- $F_{UT,T}^{\sin(\phi-\phi_S)}$, $F_{UT}^{\sin(\phi+\phi_S)}$ $\rightsquigarrow$ twist three (Qiu, Sterman)

The low-q_T calculation

factorization at twist-two accuracy (oversimplified)

$$F_{\dots} \propto \sum_i \int d^2\boldsymbol{p}_T \, d^2\boldsymbol{k}_T \, d^2\boldsymbol{l}_T \, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T) \times f^i(x, p_T^2) \, D^i(z, k_T^2) \, U(l_T^2)$$

Collins, Soper '81; Ji, Ma, Yuan '04; Collins, Rogers, Stasto '07



- $U(l_T^2)$ = soft factor
- for twist three mainly tree-level calculations
  - no detailed investigation of soft gluons
  - results similar to twist two formula w/o U

Mulders, Tangerman '97; Boer, Mulders, Pijlman '03

- in intermediate region $M \ll q_T \ll Q$ have at least one large transv. momentum: p_T or k_T or $l_T \gg M$

p_T dependent parton distributions

- matrix elements

$$\Phi^{[\Gamma]}(x, \boldsymbol{p}_T) \propto \int d\xi^- d^2\boldsymbol{\xi}_T \, e^{ip\cdot\xi} \, \langle P|\bar{\psi}(0)\,\mathcal{U}_{(0,\xi)}\,\Gamma\,\psi(\xi)|P\rangle \,\big|_{\xi^+=0}$$

with $\mathcal{U}_{(0,\xi)} =$ Wilson line from 0 to ξ along suitable path

- for unpolarized proton

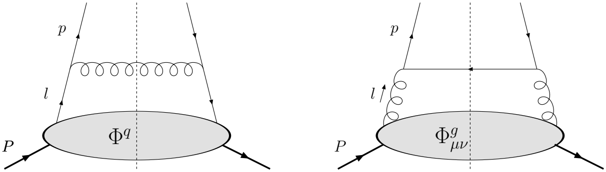
$\Gamma = \gamma^+$	$\Phi^{[\Gamma]} \propto f_1$	usual twist-two dist., unpolarized quarks
$= \gamma_T^\alpha$	$\propto \frac{p_T^\alpha}{P^+} f^\perp$	twist three
$= \gamma^+\gamma_T^\alpha\gamma_5$	$\propto \frac{\epsilon_T^{\alpha\rho} p_{T\rho}}{M} h_1^\perp$	twist two, transversely pol. quarks
$= \ldots$		

- analogous for k_T dep't fragmentation functions:

$$D_1 \leftrightarrow f_1, \quad D^\perp \leftrightarrow f^\perp, \quad H_1^\perp \leftrightarrow h_1^\perp$$

Parton densities at high p_T

- for $p_T \gg M$ can calculate matrix element $\Phi^{[\Gamma]}$ using collinear factorization
  similar to e.g. jet production in $\gamma^* p$
  factorization not proven;
    will probably fail at higher twists, i.e. high powers of $1/p_T$



Parton densities at high p_T

- power counting in $1/p_T \oplus$ Lorentz invariance
  $\oplus$ chirality conservation $\oplus$ time reversal constraints
  $\rightsquigarrow$ power behavior of $\Phi^{[\Gamma]}$

$$f_1(x, p_T^2) \sim \frac{1}{p_T^2}\, \alpha_s \sum_{\text{partons } j} [K_1^j \otimes f_1^j]$$

$$f^{\perp}(x, p_T^2) \sim \frac{1}{p_T^2}\, \alpha_s \sum_{\text{partons } j} [K^{\perp i} \otimes f_1^i]$$

$$h_1^{\perp}(x, p_T^2) \sim \frac{M^2}{p_T^4}\, \alpha_s \sum_{\text{partons } j} [K_3^j \otimes \text{collinear twist-three distributions}]$$

  with hard-scattering kernels K_1, $K^{\perp}$, K_3

- analogous for fragmentation functions

Low-q_T results in the perturbative regime of q_T

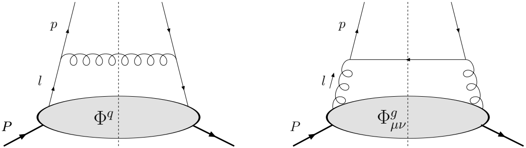
- plug results into factorization formula for struct. fcts.
  $\rightsquigarrow$ get $F_{\ldots}$ for $M \ll q_T \ll Q$
- general structure:

$$F_{\ldots} \propto \sum_{i=q,\bar{q}} \int d^2\boldsymbol{p}_T\, d^2\boldsymbol{k}_T\, d^2\boldsymbol{l}_T\, \delta^{(2)}(\boldsymbol{p}_T - \boldsymbol{k}_T + \boldsymbol{l}_T + \boldsymbol{q}_T) \times f^i(x, p_T^2)\, D^i(z, k_T^2)\, U(l_T^2)$$

$$\approx \frac{M^{k-2}}{q_T^k} \sum_{\substack{i=q,\bar{q}\\ j=q,\bar{q},g}} \Big[\big(K^{ji} \otimes f^j\big)(x)\, D^i(z) + f^i(x)\, \big(D^j \otimes L^{ji}\big)(z) + C f^i(x)\, D^i(z) \Big]$$

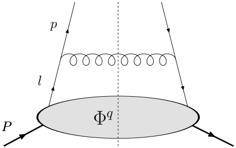
- term with coefficient C from soft factor U at $l_T \sim q_T$

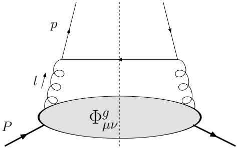
Different types of evolution



1. p_T integrated distributions
   - $f_1(x, p_T^2) \sim 1/p_T^2$ for large p_T
     $\rightsquigarrow f(x) = \int dp_T^2\, f_1(x, p_T^2)$ logarithmically divergent
   - must regularize
     heuristically: restrict integral to $p_T < \mu$
     technically: use dimensional regularization $\rightsquigarrow$ $\overline{\text{MS}}$
   - $\rightsquigarrow$ DGLAP evolution for $f_1(x; \mu^2)$
     kernels for evolution and for high-p_T behavior closely related:
     $\mu^2 \frac{d}{d\mu^2} f(x; \mu^2) \sim \mu^2 \frac{d}{d\mu^2} \int^{\mu^2} dp_T^2\, f(x, p_T^2) = \mu^2 f(x, p_T^2 = \mu^2)$
     $= \alpha_s \sum_j \big(K^j \otimes f^j\big)(x; \mu^2)$

Different types of evolution





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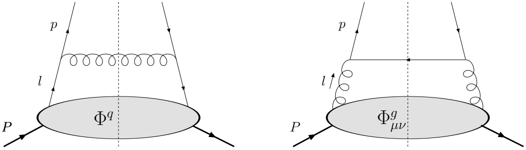
2. p_T moments, e.g.
   - $h_1^\perp(x, p_T^2) \sim 1/p_T^4$ for large p_T

     $\rightsquigarrow h_1^{(1)\perp}(x) = \int dp_T^2\, \frac{p_T^2}{2M^2}\, h_1^\perp(x, p_T^2)$ log. divergent
   - $\rightsquigarrow$ DGLAP-type evolution for $h_1^{(1)\perp}(x; \mu)$

Different types of evolution



3. p_T dependent distributions
   - no DGLAP-type evolution
   - divergences for $(l-p)^+ \to 0$ cancel in p_T integrated case

     then $(l-p)^- = \frac{\boldsymbol{p}_T^2}{2(l-p)^+} \to \infty$

     $\rightsquigarrow$ gluon moves fast to the left

     $\rightsquigarrow$ shouldn't be in parton density of right-moving proton
   - requires cutoff ζ in gluon rapidity

     $\rightsquigarrow$ Collins-Soper equation for $f_1(x, p_T^2; \zeta)$

     $\rightsquigarrow$ Sudakov factor, resum logarithms of $p_T/\zeta \sim p_T^2/Q^2$
   - analogous for $h_1^\perp(x, p_T^2; \zeta)$, $H_1^\perp(z, k_T^2; \zeta)$, ...

Compare high- and low-q_T results in region $M \ll q_T \ll Q$

	low-q_T calc.		high-q_T calc.
$F_{UU,T} \sim$	$\frac{1}{q_T^2}$	from $f_1(x, p_T^2), D_1(z, k_T^2)$	$\frac{1}{q_T^2}$
$F_{UU,L} \sim$	$\frac{1}{Q^2}$	from twist four: unknown	$\frac{1}{Q^2}$
$F_{UU}^{\cos 2\phi} \sim$	$\frac{M^2}{q_T^4}$	from $h_1^\perp, H_1^\perp$	$\frac{1}{Q^2}$
	$+\frac{1}{Q^2}$	from twist four: unknown	
$F_{UU}^{\cos \phi} \sim$	$\frac{1}{Q\, q_T}$	from $f_1, f^\perp, D_1, D^\perp$	$\frac{1}{Q\, q_T}$
$F_{UT,T}^{\sin(\phi-\phi_S)} \sim$	$\frac{M}{q_T^3}$	from $f_{1T}^\perp, D_1$	$\frac{M}{q_T^3}$

A closer look at different cases

	low-q_T calc.	high-q_T calc.
$F_{UU,T} \sim$	$\frac{1}{q_T^2}$ from $f_1(x,p_T^2), D_1(z,k_T^2)$	$\frac{1}{q_T^2}$

F_T: results exactly coincide

Collins, Soper, Sterman '85 and later work

- should not add contributions (double counting)
- in phenomenology often use Gaussian for p_T dependence of distribution functions
  $\rightsquigarrow$ lacks perturbative high-p_T tail
    $\rightsquigarrow$ no double counting when add high-q_T result
    but not a systematic procedure
    unclear how good for intermediate q_T

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F_T: results exactly coincide

Collins, Soper, Sterman '85 and later work

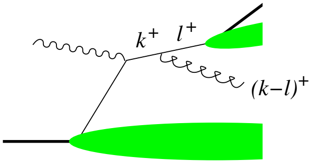
- different schemes to interpolate smoothly between the two in literature mainly used for HERA collider data and Drell-Yan

  P. Nadolsky, C.-P. Yuan et al.
- use solution of Collins-Soper equation (from low-q_T calc.) to resum $\log q_T^2/Q^2$ terms
- analogous for F_{LL} with $f_1 \to g_1$

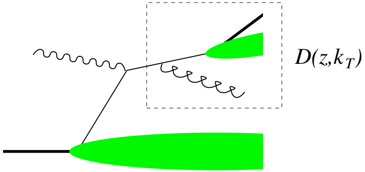
  Y. Koike, J. Nagashima, W. Vogelsang '06

Correspondence at level of graphs

high-q_T calculation

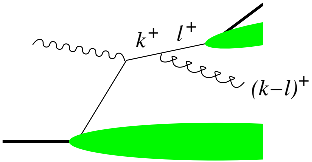


low-q_T calculation with $q_T \gg M$

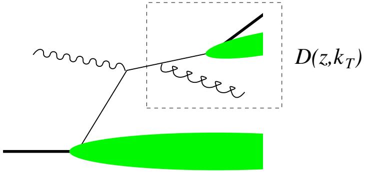


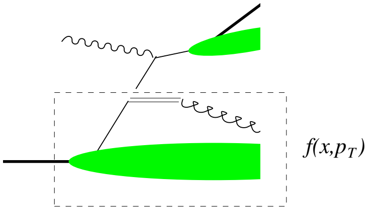
Correspondence at level of graphs

high-q_T calculation



low-q_T calculation with $q_T \gg M$





	low-q_T calc.	high-q_T calc.
$F_{UT,T}^{\sin(\phi-\phi_S)} \sim$	$\frac{M}{q_T^3}$ from $f_{1T}^{\perp}, D_1$	$\frac{M}{q_T^3}$

$F_{UT,T}^{\sin(\phi-\phi_S)}$: also exact agreement

- requires twist-three calculation at high q_T

  X. Ji, J.-W. Qiu, W. Vogelsang, F. Yuan '06
  Y. Koike, W. Vogelsang, F. Yuan '07

- expect same situation for Collins asymmetry $F_{UT,T}^{\sin(\phi+\phi_S)}$
  power counting clear, but explicit calculation not done

	low-q_T calc.		high-q_T calc.
$F_{UU}^{\cos\phi} \sim$	$\frac{1}{Q\,q_T}$	from $f_1, f^\perp, D_1, D^\perp$	$\frac{1}{Q\,q_T}$

$F_{UU}^{\cos\phi}$: do not have complete twist-three result for low q_T

- if take soft factor from low-q_T twist-two formula (working assumption) then result agree except for term $\propto f_1(x)\,D_1(z)$
- $\rightsquigarrow$ soft-gluons need special attention for establishing twist-three factorization
- this is required if want to use Collins-Soper-Sterman method to resum $\log q_T^2/Q^2$ terms

	low-q_T calc.	high-q_T calc.
$F_{UU,L} \sim$	$\frac{1}{Q^2}$ from twist four: unknown	$\frac{1}{Q^2}$

F_L:

- twist four calc. for SIDIS at low q_T well beyond reach
  strong doubts whether factorization actually holds
- calculation in parton model result involving $f_1(x, p_T^2)\, D_1(z, k_T^2)$
  has power behavior $\sim 1/Q^2$
  but differs from high-q_T result at intermediate q_T

	low-q_T calc.		high-q_T calc.
$F_{UU}^{\cos 2\phi} \sim$	$\frac{M^2}{q_T^4}$	from $h_1^\perp, H_1^\perp$	
	$+\frac{1}{Q^2}$	from twist four: unknown	$\frac{1}{Q^2}$

$F_{UU}^{\cos 2\phi}$:

- high-q_T result should match with unknown twist-four result at low q_T
- at low q_T have leading contrib'n with $h_1^\perp(x, p_T^2)\, H_1^\perp(z, k_T^2)$ should match with unknown twist-four expression at high q_T
- leading contrib's from both high and low q_T important for $M \ll q_T \ll Q$
  can add them without double counting

	low-q_T calc.		high-q_T calc.
$F_{UU}^{\cos 2\phi} \sim$	$\frac{M^2}{q_T^4}$	from $h_1^\perp, H_1^\perp$	
	$+\frac{1}{Q^2}$	from twist four: unknown	$\frac{1}{Q^2}$

$F_{UU}^{\cos 2\phi}$: interpolating from low to high q_T

- for asymmetry $A_{UU}^{\cos 2\phi} = \dfrac{\varepsilon F_{UU}^{\cos 2\phi}}{F_{UU,T} + \varepsilon F_{UU,L}}$ can use

$$A_{UU}^{\cos 2\phi} \approx \frac{\varepsilon L_{UU}^{\cos 2\phi}}{L_{UU,T}} + \frac{\varepsilon H_{UU}^{\cos 2\phi}}{H_{UU,T} + \varepsilon H_{UU,L}} \quad \text{at all } q_T$$

in regions where leading results $L_{\dots}$ at low or $H_{\dots}$ at high q_T are not valid, they give contrib's suppressed by M^2/q_T^2 or q_T^2/Q^2

Integrated observables

introduce shorthand notation

$$\left\langle\!\!\left\langle \left(\frac{q_T}{M}\right)^p F(Q,q_T) \right\rangle\!\!\right\rangle \overset{\text{def}}{=} \pi z^2 \int_0^{q_{\text{max}}^2} dq_T^2 \left(\frac{q_T}{M}\right)^p F(Q,q_T)$$

split integration into

$q_T^2 < \Gamma M^2$	low q_T	
$\Gamma M^2 < q_T^2 < \gamma Q^2$	intermediate q_T	$\Gamma \gg 1$, $\gamma \ll 1$
$\gamma Q^2 < q_T^2 < q_{\text{max}}^2$	high q_T	

- with suitable weight factor $(q_T/M)^p$ achieve deconvolution of p_T integrals in low-q_T results
- using results for power behavior in different regions can establish their relative importance in integral

$F_{UU,T}$ revisited

	low q_T	intermediate q_T	high q_T
$f(q_T)$	$\int_0^{\Gamma M^2} dq_T^2\, f(q_T)$	$\int_{\Gamma M^2}^{\gamma Q^2} dq_T^2\, f(q_T)$	$\int_{\gamma Q^2}^{q^2_{\text{max}}} dq_T^2\, f(q_T)$
$F_{UU,T}$	$\ln \Gamma$	$\ln \left[\dfrac{\gamma}{\Gamma} \, \dfrac{Q^2}{M^2} \right]$	$\ln \dfrac{1}{\gamma}$

- all regions contribute at same power $\rightsquigarrow$ how to join?
- heuristically: use low-q_T result for $q_T < \mu$
  and high-q_T result for $q_T > \mu$

$$\rightsquigarrow \quad \langle\!\langle F_{UU,T} \rangle\!\rangle = \sum_j x\, e_j^2 \; f_1^j(x;\mu^2)\, D_1^j(z;\mu^2) + \{\alpha_s \text{ term}\}$$

  technically: dimensional regularization
- choice $\mu \sim Q$ avoids large $\log(Q^2/\mu^2)$ terms in α_s term
- $\rightsquigarrow$ relate factorization for q_T dependent and q_T integrated observables

$F_{UU,L}$ revisited

$f(q_T)$	low q_T $\int_0^{\Gamma M^2} dq_T^2\, f(q_T)$	intermediate q_T $\int_{\Gamma M^2}^{\gamma Q^2} dq_T^2\, f(q_T)$	high q_T $\int_{\gamma Q^2}^{q^2_{\mathrm{max}}} dq_T^2\, f(q_T)$
$F_{UU,L}$	$\frac{M^2}{Q^2}\,\Gamma$	γ	1

- low-q_T region gives power suppressed contribution
- $\rightsquigarrow$ $\langle\!\langle F_{UU,L} \rangle\!\rangle = \mathcal{O}(\alpha_s)$

Sivers and Collins asymmetries

	low q_T	intermediate q_T	high q_T
$F_{UT,T}^{\sin(\phi-\phi_S)}$, $F_{UT}^{\sin(\phi+\phi_S)}$	1	$\dfrac{1}{\sqrt{\Gamma}}$	$\dfrac{1}{\sqrt{\gamma}}\,\dfrac{M}{Q}$
$\dfrac{q_T}{M}\,F_{UT,T}^{\sin(\phi-\phi_S)}$, $\dfrac{q_T}{M}\,F_{UT}^{\sin(\phi+\phi_S)}$	$\ln\Gamma$	$\ln\left[\dfrac{\gamma}{\Gamma}\,\dfrac{Q^2}{M^2}\right]$	$\ln\dfrac{1}{\gamma}$

- unweighted integrals: dominated by low q_T, convolutions in p_T
- weighted integrals: all regions contribute at same power
  similarly to case of $\langle\!\langle F_{UU,T} \rangle\!\rangle$ get

$$\langle\!\langle (q_T/M)\, F_{UT,T}^{\sin(\phi-\phi_S)} \rangle\!\rangle = -2 \sum_j x\, e_j^2\; f_{1T}^{j\perp(1)}(x;Q^2)\, D_1^j(z;Q^2) + \{\alpha_s \text{ term}\}$$

- DGLAP kernel for $f_{1T}^{j\perp(1)}(x;Q^2)$ essentially known
  from known high-p_T behavior of $f_{1T}^{j\perp}$
- part of α_s term known from twist-three calc. of H. Eguchi et al. '06

Angular asymmetries

	low q_T	intermediate q_T	high q_T
$F_{UU}^{\cos\phi}$	$\frac{M}{Q}\sqrt{\Gamma}$	$\sqrt{\gamma}$	1
$\frac{q_T}{M} F_{UU}^{\cos\phi}$	$\frac{M}{Q}\,\Gamma$	$\gamma\,\frac{Q}{M}$	$\frac{Q}{M}$

- both weighted and unweighted integral dominated by high q_T $\rightsquigarrow$ not suited for studying low-q_T twist-three result
- up to corrections of order M^2/Q^2 have $\langle\!\langle (q_T/M) F_{UU}^{\cos\phi} \rangle\!\rangle = \mathcal{O}(\alpha_s)$ just like $\langle\!\langle F_{UU,L} \rangle\!\rangle$
- might use this to obtain information on (usual collinear) parton densities and fragmentation fct's (gluon enhanced)
- fully analogous for $\langle\!\langle (q_T/M) F_{LL}^{\cos\phi} \rangle\!\rangle$ with polarized densities

		low q_T	intermediate q_T	high q_T
$F_{UU}^{\cos 2\phi}$	(low q_T)	1	$\dfrac{1}{\Gamma}$	
	(high q_T)		γ	1
$\dfrac{q_T^2}{M^2}\, F_{UU}^{\cos 2\phi}$	(low q_T)	$\ln\Gamma$	$\ln\left[\dfrac{\gamma}{\Gamma}\,\dfrac{Q^2}{M^2}\right]$	
	(high q_T)		$\gamma^2\,\dfrac{Q^2}{M^2}$	$\dfrac{Q^2}{M^2}$

- weighted integral: dominated by high q_T
  $\rightsquigarrow$ not suited for studying Boer-Mulders and Collins fct.
- unweighted integral: leading contr's from both low and high q_T
  - for latter need $f_1^{q,g}$ and $D_1^{q,g}$ $\rightsquigarrow$ information on $h_1^\perp$ and $H_1^\perp$
  - otherwise take differential $F_{UU}^{\cos 2\phi}(Q, q_T)$ or integral with upper cutoff

  $\rightsquigarrow$ (unfortunately) no deconvolution of p_T integrals
- analogous situation for $\cos 2\phi$ asymmetries in e^+e^- (Belle) and in Drell-Yan

Summary

- high-p_T behavior of parton densities/fragmentation fcts. from perturbation theory
  $\rightsquigarrow$ join descriptions for low and high q_T in intermediate region
- leading results for high and low q_T match for some observables but not for others
  $\rightsquigarrow$ different strategies if want to describe all q_T
- integrated obs' can be dominated by low or high q_T or by both
  $\rightsquigarrow$ different information on hadron structure
- weighted asymmetries
  - good for Sivers and Collins
    prospect of full NLO calculation
  - not good for Boer Mulders $\langle\!\langle (q_T/M) F_{UU}^{\cos 2\phi} \rangle\!\rangle$
  - may be useful for collinear distributions
    $\langle\!\langle (q_T/M) F_{UU}^{\cos \phi} \rangle\!\rangle$, $\langle\!\langle (q_T/M) F_{LL}^{\cos \phi} \rangle\!\rangle$, similar to $\langle\!\langle F_{UU,L} \rangle\!\rangle$